

8) $f = (x^3 - 5)(x^5 - 7)$, $K = \text{splitting field}$, $n = [K:\mathbb{Q}]$

a) $\exists \alpha, \beta \in K$ s.t. $\mathbb{Q}(\alpha), \mathbb{Q}(\beta) \subseteq K$, α a root of g
 β " " " " h .

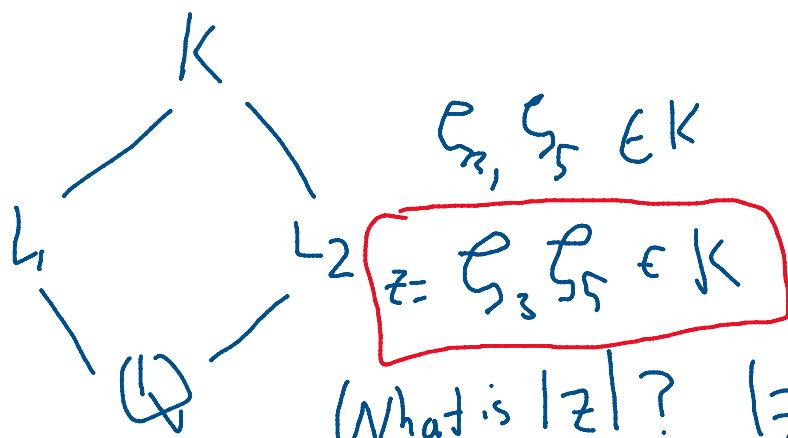
b) L_1 , the splitting field of g :

$L_1 = \mathbb{Q}(\sqrt[3]{5}, \zeta_3)$ ← deg 6 b/c rel prime
 $\subseteq \checkmark$
 $\Rightarrow [\mathbb{Q}(\sqrt[3]{5}) : \mathbb{Q}] = 3$
 $[\mathbb{Q}(\zeta_3) : \mathbb{Q}] = \phi(3) = 2$

L_2 splitting field of h

$L_2 = \mathbb{Q}(\sqrt[5]{7}, \zeta_5)$ $\text{deg}/\mathbb{Q} = 20$

$K \subseteq L_1 L_2 = \mathbb{Q}(\sqrt[3]{5}, \sqrt[5]{7}, \zeta_3, \zeta_5)$



What is $|z|$? $|z| = 15$, satisfying $\Phi_{15}(x)$
 $\text{min. p.c.} = \phi(15)$

Ψ What is $|Z|$? $|Z|=1$, $\text{deg } \phi_{15} = \phi(15) = 8$

Claim: $K = \mathbb{Q}(\sqrt[3]{5}, \sqrt[5]{7}, z)$

$$\mathbb{Q}(\sqrt[3]{5}, \sqrt[5]{7}) \subseteq K \subseteq \mathbb{Q}(\sqrt[3]{5}, \sqrt[5]{7}, z)$$

$\underbrace{\hspace{10em}}_{(1,2,4)}$

But ϕ_3 is not reducible over $\mathbb{Q}(\sqrt[3]{5}, \sqrt[5]{7}, \rho_5)$

$$\Leftrightarrow \mathbb{Q}(z_3, \rho_5) \neq \mathbb{Q}(\rho_5)$$

So must have $K = \mathbb{Q}(\sqrt[3]{5}, \sqrt[5]{7}, z)$
 $n=120$