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More Integration by Parts

• Work 1d from worksheet 1A

You saw on worksheet 1A that sometimes we may need to do integration by parts more than once.

ex 1) Integrate $g(x) = x^2 e^{2x}$.

$$\int x^2 e^{2x} = \frac{1}{2}x^2 e^{2x} - \int x e^{2x}\,dx \qquad \begin{array}{ll} u = x^2 & dv = e^{2x}\,dx \\ du = 2x\,dx & v = \frac{1}{2}e^{2x} \end{array}$$

$$= \frac{1}{2}x^2 e^{2x} - \left(\frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x}\,dx\right) \qquad \begin{array}{ll} u = x & dv = e^{2x}\,dx \\ du = dx & v = \frac{1}{2}e^{2x} \end{array}$$

$$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C$$

• discuss why swapping u and v results in undoing work

ex 2) Cyclic by parts: Find $\int e^x \cos(x)\,dx$. (see ex. 3 in 7.1 of text)

Note: either choice works.

$$(1) \quad \int \underbrace{e^x}_{u} \underbrace{\cos(x)\,dx}_{dv} = \underbrace{e^x}_{u}\underbrace{\sin(x)}_{v} - \int \underbrace{\sin(x)}_{v}\underbrace{e^x\,dx}_{du}$$

$$(2) \quad \int \sin(x) e^x\,dx = -e^x\cos(x) + \underline{\underline{\int e^x \cos(x)\,dx}} \qquad \begin{array}{ll} u = e^x & dv = \sin(x)\,dx \\ du = e^x\,dx & v = -\cos(x) \end{array}$$

Got the original integral back! We can now substitute (2) into (1) and solve:

$$\int e^x \cos(x)\,dx = e^x\sin(x) - \left(-e^x\cos(x) + \int e^x\cos(x)\,dx\right)$$

$$= e^x\sin(x) + e^x\cos(x) - \int e^x\cos(x)\,dx$$

$$2\int e^x\cos(x)\,dx = e^x(\sin(x) + \cos(x))$$

$$\int e^x\cos(x)\,dx = \boxed{\frac{1}{2}e^x(\sin(x) + \cos(x)) + C}$$

ex 3) Definite integral: Find $\int_1^3 \ln(x)\,dx$.

$$\int_1^3 \ln(x)\,dx = x\ln(x)\Big|_1^3 - \int_1^3 \overset{x|_1^3 =}{1}\,dx \qquad \begin{array}{ll} u = \ln(x) & dv = dx \\ du = \frac{1}{x}dx & v = x \end{array}$$

$$= 3\ln(3) - 1\,\cancel{\ln(1)}^{\,0} - (3-1)$$

$$= \boxed{3\ln(3) - 2}$$