Summer II 2018 - Calc 2

6/7/18

- Syllabus / About you survey
- Review worksheet
  - answer on board 1-2 problems (did 4c, d, 5)

Advanced u-sub ex:

Find $\int x\sqrt{2x+1}\,dx$. Let $u=2x+1$, $du=2\,dx$

Then: $\int x\sqrt{2x+1}\,dx = \int x\sqrt{u}\,\frac{du}{2}$

- need to have only u, so let's write x in terms of u

$u=2x+1 \Rightarrow x=\frac{1}{2}(u-1)$

$$= \frac{1}{4}\int (u-1)\sqrt{u}\,du$$
$$= \frac{1}{4}\int (u-1)u^{1/2}\,du$$
$$= \frac{1}{4}\int u^{3/2}-u^{1/2}\,du$$
$$= \frac{1}{4}\left(\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right)+C$$
$$= \frac{1}{10}u^{5/2}-\frac{1}{6}u^{3/2}+C$$
$$= \frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}+C$$

§ 7.1: Integration by Parts

Big idea: We used u-sub to undo the chain rule. Now we will undo the product rule:

$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x)+f(x)g'(x)$$

Integration by parts tells us that

$$\int f(x)g'(x)\,dx = f(x)g(x)-\int f'(x)g(x)\,dx$$ (Rearrange the above and integrate)

other notation: $\boxed{\int u\,dv = uv - \int v\,du}$

Ex 1: Compute $\int xe^x\,dx$. (compare exercise 1 in 7.1 of text)

Choose $u=x$ $dv=e^x\,dx$

$du=dx$ $v=e^x$

$$\int \underset{u\cdot v}{xe^x\,dx} = \underset{u\cdot v}{xe^x} - \int \underset{v\cdot du}{e^x\,dx}$$
$$= xe^x-e^x+C$$

What if we chose $u=e^x$ $dv=x\,dx$

$du=e^x\,dx$ $v=\frac{1}{2}x^2$

$$\int xe^x\,dx = \underset{u\cdot v}{\frac{1}{2}x^2e^x} - \underbrace{\int \frac{1}{2}x^2e^x\,dx}_{\text{harder. not good!}}$$

Check: $\frac{d}{dx}(xe^x-e^x+C) = e^x+xe^x-e^x$

$= xe^x$ ✓

Ex 2 Compute $\int x\cos(x)\,dx$.

$u = x$ $\quad dv = \cos(x)\,dx$

$du = dx$ $\quad v = \sin(x)$

$$\int x\cos(x)\,dx = x\sin(x) - \int \sin(x)\,dx$$

$$= \boxed{x\sin(x) + \cos(x) + C}$$

Ex 3 Find an antiderivative of $y = \arctan(x)$. (see ex 2 in 7.1 of text)

$$\int \arctan(x)\,dx$$

$u = \arctan(x)$ $\quad dv = dx$

$du = \frac{1}{1+x^2}\,dx$ $\quad v = x$

$$= x\arctan(x) - \int \frac{x}{1+x^2}\,dx$$

Substitute $s = 1+x^2$ $\quad ds = 2x\,dx \Rightarrow dx = \frac{ds}{2x}$

$$= x\arctan(x) - \int \frac{x}{s}\,\frac{ds}{2x}$$

$$= x\arctan(x) - \frac{1}{2}\int \frac{1}{s}\,ds$$

$$= x\arctan(x) - \frac{1}{2}\ln|s| + C$$

$$= \boxed{x\arctan(x) - \frac{1}{2}\ln|x^2+1| + C}$$

- Worksheet 1A