Absolute vs Conditional Convergence

A series $\sum_{n=1}^{\infty} a_n$ <u>converges absolutely</u> if $\sum_{n=1}^{\infty} |a_n|$ converges. If $\sum_{n=1}^{\infty} |a_n|$ diverges, but $\sum_{n=1}^{\infty} a_n$ converges, then we say the series <u>converges conditionally</u>.

<u>e.g.</u>: $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ converges conditionally since we saw $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ converges by AST but $\sum_{n=1}^{\infty} \left|(-1)^{n-1} \frac{1}{n}\right| = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

<u>ex</u>: $\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{n^{3/2}}$ converges absolutely since $\sum_{n=3}^{\infty} \left|\frac{(-1)^{n-1}}{n^{3/2}}\right| = \sum_{n=3}^{\infty} \frac{1}{n^{3/2}}$ is a convergent p-series

<u>Notes</u>:

1) Any absolutely convergent series is also convergent: being absolutely convergent is stronger
2) Conditionally convergent series are weird. If you rearrange their terms, the value of the series ~~changes~~ can change. (see Desmos ex)
3) Roughly speaking, conditional convergence means the series is only converging b/c of cancellations btwn +/- terms.

<u>Ex</u>: $\sum_{n=1}^{\infty} \sin(n) \left(\frac{2^n - 1}{5^n + n}\right)$. $\left|\sin(n)\left(\frac{2^n-1}{5^n+n}\right)\right| \leq \frac{2^n - 1}{5^n + n} \leq \frac{2^n}{5^n + n} \leq \left(\frac{2}{5}\right)^n$,

so $\sum_{n=1}^{\infty} \left|\sin(n)\left(\frac{2^n-1}{5^n+n}\right)\right|$ converges by CT. Thus $\sum_{n=1}^{\infty} \sin(n)\left(\frac{2^n-1}{5^n+n}\right)$ is <u>abs conv.</u>

<u>Class examples</u>: