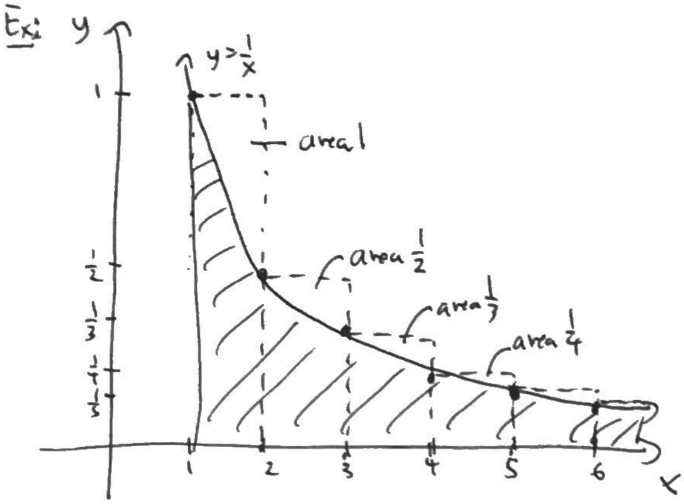
6/25/18 Lecture - 11.3 Integral Test

• Begin with quick recap of Friday, Q from WeBWorK Worksheet 9a (geom series, telescoping, partial sums, div test)

Last time, we said $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Let's see why:

Ex:



$/// = \int_1^{\infty} \frac{1}{x}\,dx \Rightarrow \infty$

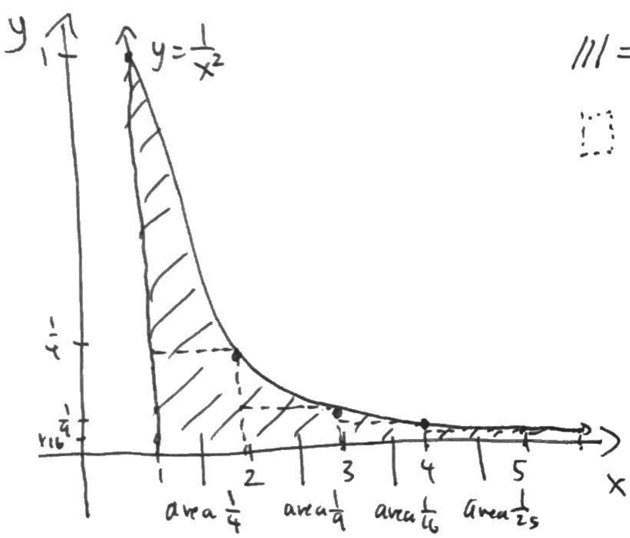
$[\,] = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n}.$

Notice that the area of the rectangles is greater than the area under the graph, which is infinite.

So $\sum_{n=1}^{\infty} \frac{1}{n}$ must also be infinite.

The idea of comparing to an integral is also useful for convergent series:

Ex: Consider $\sum_{n=1}^{\infty} \frac{1}{n^2}$:



$/// = \int_1^{\infty} \frac{1}{x^2}\,dx = 1$

$[\,] = \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots = \sum_{n=2}^{\infty} \frac{1}{n^2}$

From the picture, $\sum_{n=2}^{\infty} \frac{1}{n^2} < \int_1^{\infty} \frac{1}{x^2}\,dx$.

So $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \sum_{n=2}^{\infty} \frac{1}{n^2} < 1 + \int_1^{\infty} \frac{1}{x^2}\,dx = 1+1=2.$

So $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

Notes: • $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, but this argument does not tell us what it converges to. (in fact $\frac{\pi^2}{6}$) (by Euler & others)

• This also illustrates the useful idea that only the tail of a series matters for convergence.

We ignored the n=1 term at first and added it back after we found that the other terms converged.

Integral Test: Consider $\sum_{n=k}^{\infty} a_n$. If $a_n = f(n)$, where f is cts, positive, and decreasing (eventually), then ~~If $a_n \geq 0$ and $a_{n+1} \leq a_n$ eventually, then and $a_n = f(n)$, where f is cts,~~

$\sum_{n=k}^{\infty} a_n$ converges if and only if $\int_k^{\infty} f(x)\,dx$ converges.