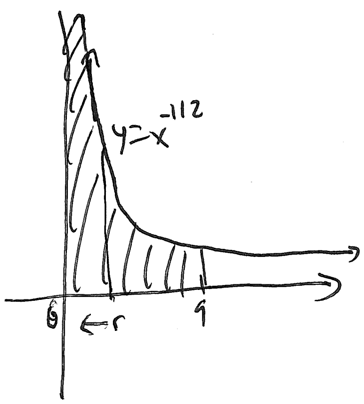
§7.8 Improper Integrals Part 2

We can also ask for the area under the graph of $f(x)$ on an interval where f has a discontinuity:

Ex: $\displaystyle\int_0^9 \frac{dx}{\sqrt{x}} = \lim_{r\to 0^+} \int_r^9 \frac{dx}{\sqrt{x}} = \lim_{r\to 0^+} 2\sqrt{x}\Big|_r^9 = \lim_{r\to 0^+} 6 - 2\sqrt{r} = 6$, so the integral converges

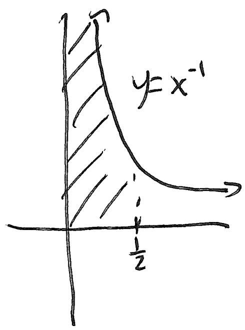


Ex $\displaystyle\int_0^{1/2} \frac{dx}{x} = \lim_{r\to 0^+} \int_r^{1/2} \frac{dx}{x} = \lim_{r\to 0^+} \ln|x|\Big|_r^{1/2}$

$\displaystyle = \lim_{r\to 0^+} \ln|1/2| - \ln|r|$

$= \infty$,

so the integral diverges.



Comparing Integrals

Sometimes, we want to know if an improper integral converges, but can't evaluate it.

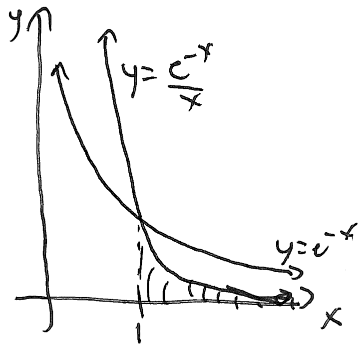
ex: $\displaystyle\int_1^\infty \frac{e^{-x}}{x}\,dx$

But: if $x \geq 1$, $0 \leq \frac{1}{x} \leq 1$, so $0 \leq \frac{e^{-x}}{x} \leq e^{-x}$,

i.e. $y = e^{-x}/x$ lies underneath $y = e^{-x}$ for $x \geq 1$.

So, $\displaystyle 0 \leq \int_1^\infty \frac{e^{-x}}{x}dx \leq \int_1^\infty e^{-x}\,dx = e^{-1}$ (Friday)

So $\displaystyle\int_1^\infty \frac{e^{-x}}{x}dx$ converges.



Thm: Let $f(x) \geq g(x) \geq 0$ for $x \geq a$.

a) If $\int_a^\infty f(x)\,dx$ converges, then $\int_a^\infty g(x)dx$ converges

b) If $\int_a^\infty g(x)\,dx$ diverges, then $\int_a^\infty f(x)dx$ diverges.

ex: $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$: $\displaystyle\frac{1}{\sqrt{x^3+1}} \leq \frac{1}{\sqrt{x^3}}$ (since denom is larger)

- $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3}}dx$ converges (p-integral with $p = 3/2 > 1$)
- So by the comparison theorem, $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$ converges.