10.1 Parametric Curves

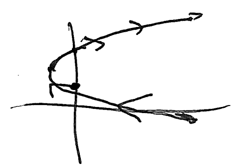
<u>Motivation</u>: Suppose we want to describe the motion of an object through space even if the curve it travels is not a function:

One approach is to describe the (x,y) position as functions of time $x(t)$, $y(t)$ or
$x=f(t)$, $y=g(t)$.

These are called <u>parametric curves</u>. Parametric curves always have an orientation – the direction that the parameter increases.

| (t^3, t^6), (t^2, t^4), $(\cos t, \cos^2 t)$

Ex: $x=t^2-2t$, $y=t+1$



Ex: $x=t^2-2t$, $y=t+1$, $0\le t\le 4$.

- The same curve can be represented by multiple parameterizations:
  $x=\cos t$, $y=\sin t$, $0\le t\le 2\pi$ and $x=\cos 2t$, $y=\sin 2t$, $0\le t\le 2\pi$
- Thus we distinguish a <u>curve</u> (a set of points) from a <u>parametric curve</u> (where the points are traced in a particular way.

It is useful to be able to convert between parametric and Cartesian equations for a given curve.

Ex: $x=2t-1$, $y=\frac{1}{2}t+1$

Ex: $x=t^2$, $y=\ln t$

Ex: $x=\sqrt{t+1}$, $y=\sqrt{t-1}$

Ex: Find parameterizations for
1) $(x-h)^2+(y-k)^2=r^2$ (circle of radius r at (h,k))
2) $y=x^2$ from $x=1$ to $x=5$
3) $x^2=y^3$ from $x=0$ to $x=4$

Tomorrow, we will begin investigating how to find familiar quantities like tangents, areas, arc lengths, and surface areas for parametric curves.