Average Value:

- How do we find the average velocity of an object?

$$v_{avg} = \frac{\Delta x}{\Delta t}.$$

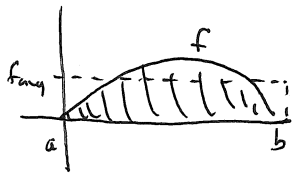
When v is a function of time, this becomes $v_{avg} = \dfrac{\int_a^b v(t)\,dt}{b-a}$.

This works generally: $f_{avg} = \dfrac{1}{b-a}\int_a^b f(t)\,dt$ for t in $[a,b]$.

Alt:

$\int_a^b f(x)\,dx$ is the area under f on $[a,b]$.

f_{avg} is the height of the rectangle w/ same area:



ex: Find the average value of $f(x) = \sin(x)$ on $[0,\pi]$.

$$f_{avg} = \frac{1}{\pi}\int_0^\pi \sin(x)\,dx = -\frac{1}{\pi}\cos(x)\Big|_0^\pi = -\frac{1}{\pi}(-1) + \frac{1}{\pi}(1) = \frac{2}{\pi} \approx 0.637$$

ex: Find the average speed over the time interval $[1,5]$ of a particle whose position at time t is $s(t) = t^3 - 6t^2$ m/s.

$$s_{avg} = \frac{1}{5-1}\int_1^5 t^3 - 6t^2\,dt = \frac{1}{4}\left(\frac{t^4}{4} - 2t^3\Big|_1^5\right) = \frac{1}{4}\left(\frac{5^4}{4} - 2(5)^3 - \frac{1}{4} + 2\right) = -23 \text{ m/s}$$