Root & Ratio Tests: Series that behave "like" a geometric series.

Ratio Test: $\sum_{n=1}^{\infty} a_n$:

Let $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$. If $0 \le L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

If $L > 1$, $\sum_{n=1}^{\infty} a_n$ diverges.

If $L = 1$, the test is inconclusive.

- especially useful for: products, ~~pow~~ n^{th} powers, factorials.

ex: $\sum_{n=1}^{\infty} \frac{2^n}{n!}$. Find $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right| = \lim_{n\to\infty} \left| \frac{2}{n+1} \right| = 0$.

So the series is abs. conv. by the Ratio Test

ex: $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^2}$. Find $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-3)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-3)^n} \right| = \lim_{n\to\infty} \frac{3^{n+1}}{3^n} \cdot \frac{n^2}{(n+1)^2} = 3 \lim_{n\to\infty} \frac{n^2}{(n+1)^2} = 3$.

So the series is ~~abso~~ divergent by the Ratio Test.

ex: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2}{\sqrt{n}}$. $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{\frac{2}{\sqrt{n+1}}}{\frac{2}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} = 1$. So the Ratio Test is inconclusive.

- The ratio test ~~can~~ is usually inconclusive for series whose terms contain only powers of n.

Root Test: $\sum_{n=1}^{\infty} a_n$:

Let $L = \lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \lim_{n\to\infty} \sqrt[n]{|a_n|}$. If $0 \le L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

If $L > 1$, $\sum_{n=1}^{\infty} a_n$ diverges.

If $L = 1$, the test is inconclusive.

- useful for n^{th} powers
- always works if ratio test works, but the limit may be ugly.

ex: $\sum_{n=1}^{\infty} \left(\frac{n}{2n+3} \right)^n$