An example of a non-alternating series with $a_n \to 0$, ~~Pot~~ $a_{n+1} \le a_n$ for all $n$:

Consider the series $1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$

$$\sum_{n=1}^{\infty} a_n, \text{ where } a_n = \begin{cases} \frac{1}{n} & \text{if } n \text{ is not a multiple of } 3 \\ -\frac{1}{n} & \text{if } n \text{ is a multiple of } 3. \end{cases}$$

We will look at the partial sums $S_{3N}$ and show these diverge, hence the entire sequence of partial sums diverges.

$$S_{3N} = \sum_{k=1}^{3N} a_k = \left(1 + \frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right) + \dots + \left(\frac{1}{3N-2} + \frac{1}{3N-1} - \frac{1}{3N}\right)$$

What is $\frac{1}{3k-2} + \frac{1}{3k-1} - \frac{1}{3k}$ $= \frac{3k(3k-1) + 3k(3k-2) - (3k-1)(3k-2)}{3k(3k-1)(3k-2)}$

$$= \frac{9k^2 - 3k + 9k^2 - 6k - 9k^2 + 9k - 2}{3k(9k^2 - 9k + 2)}$$

$$= \frac{9k^2 - 2}{27k^3 - 27k^2 + 6k}.$$

So, $S_{3N} = \left(1 + \frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{3N-2} + \frac{1}{3N-1} - \frac{1}{3N}\right)$

$$= \sum_{k=1}^{N} \frac{9k^2 - 2}{27k^3 - 27k^2 + 6k}$$

But the series $\sum_{k=1}^{\infty} \frac{9k^2-2}{27k^3 - 27k^2 + 6k}$ diverges by Limit Comparison with $\sum_{k=1}^{\infty} \frac{1}{3} \cdot \frac{1}{k}$.

So the sequence of partial sums $\{S_{3N}\}$ diverges to $\infty$, hence the series

$1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$ Diverges.