Remember that for the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ to converge by AST, we must have $a_n \geq a_{n+1}$ for all n past some point.

Here's an example where $\lim_{n\to\infty} a_n = 0$, but the alternating series diverges.

Let $a_n = \begin{cases} \dfrac{1}{n/2} & \text{if } n \text{ even} \\ \dfrac{1}{\left(\frac{n+1}{2}\right)^2} & \text{if } n \text{ odd} \end{cases}$

Then $\sum_{n=1}^{\infty}(-1)^n a_n = -\frac{1}{1^2} + \frac{1}{1} - \frac{1}{2^2} + \frac{1}{2} - \frac{1}{3^2} + \frac{1}{3} - \dots$

Let's look at the even partial sums, S_{2N}.

$$S_{2N} = \sum_{k=1}^{N}\left(\frac{-1}{k^2} + \frac{1}{k}\right) = \sum_{k=1}^{N}\left(\frac{k-1}{k^2}\right)$$

One can use the limit comparison test to show that $\lim_{N\to\infty} S_{2N}$ diverges.

Since the even partial sums do not converge, the limit $\lim_{N\to\infty} S_N$ must diverge also, and so our alternating series diverges.