

MathExcel Monday Worksheet A: Integration Techniques

1. Evaluate $\int_1^e 2t^2 \ln(t) \, dt$.
2. A particle that moves along a straight line has velocity $v(t) = t^2 e^{-3t}$ meters per second after t seconds. How many meters will it travel during the first T seconds of its journey?
3. For each part, label the sides of a right triangle to reveal the geometric intuition behind an appropriate trigonometric substitution. Depending on your choice of reference angle, there are two possible correct substitutions in each case. You needn't fully evaluate each integral, although you may if you so choose.

(a) $\int \sqrt{b^2 x^2 - a^2} \, dx.$

(b) $\int \sqrt{a^2 - b^2 x^2} \, dx.$

(c) $\int \sqrt{b^2 x^2 + a^2} \, dx.$

4. Use any method or combination of methods to evaluate the following indefinite integrals.

(Recall all of the tools at your disposal: u-substitution, integration by parts, trig identity shenanigans, and trig substitution are all fair game here.)

(a) $\int \frac{1}{\sqrt{9 - x^2}} \, dx$

(e) $\int \frac{\sqrt{x^2 + 16}}{x^4} \, dx$

(b) $\int e^x \cos(x) \, dx$

(f) $\int x^2 \cos(4x) \, dx$

(c) $\int \frac{1}{\sqrt{7 - 3x^2}} \, dx$

(g) $\int \frac{x^2 - 1}{x^2 - 16} \, dx$

(d) $\int \frac{x}{\sqrt{x^2 + 100}} \, dx$

(h) $\int \cos(x) \ln(\sin(x)) \, dx$

5. (a) Use any method or combination of methods to evaluate $\int \sec(x) \, dx$.

(Ask for a hint if you get stuck.)

(b) Evaluate $\int \sqrt{x^2 + 1} \, dx$.

6. Let $a, b > 0$. Prove that the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab .