## MA 114 Worksheet \#18: Volumes I

1. If a solid has a cross-sectional area given by the function $A(x)$, what integral should be evaluated to find the volume of the solid?
2. Calculate the volume of the solid: the base is a square, one of whose sides is the interval $[0, l]$ along the $x$-axis, and the cross sections perpendicular to the $x$-axis are rectangles of height $f(x)=x^{2}$.
3. Calculate the volume of the solid whose base is the region enclosed by $y=x^{2}$ and $y=3$, where cross sections perpendicular to the $y$-axis are squares.
4. The base of a certain solid is the triangle with vertices at $(10,5),(5,5)$, and the origin. Cross-sections perpendicular to the y-axis are squares. Find the volume of the solid.
5. Calculate the volume of the solid whose base is a circle of radius $r$ centered at the origin and which has square cross sections perpendicular to the $x$-axis.
6. Calculate the volume of the following solid. The base is the parabolic region $\{(x, y) \mid$ $\left.x^{2} \leq y \leq 4\right\}$. The cross sections perpendicular to the $y$-axis are right isosceles triangles whose hypotenuse lies in the region.
7. Sketch the solid whose volume is given by the integral

$$
\pi \int_{0}^{1}\left(y^{2}+1\right)^{2}-1 d y
$$

8. For each of the following, use disks or washers to find the an integral expression for the volume of the solid obtained by rotating the given region about the specified line. Evaluate the integrals for parts (a) and (b).
(a) $R$ is the region bounded by $y=1-x^{2}$ and $y=0$; about the $x$-axis.
(b) $R$ is the region bounded by $y=1-x^{2}$ and $y=0$; about the line $y=-1$.
(c) Compare the volumes you found in parts (a) and (b). Which is bigger? Why?
(d) $R$ is the region bounded by $y=\frac{1}{x}, x=1, x=2$, and $y=0$; about the $x$-axis.
(e) $R$ is the region bounded by $x=2 \sqrt{y}, x=0$, and $y=9$; about the $y$-axis.
(f) $R$ is the region bounded by $y=e^{-x}, y=1$, and $x=2$; about the line $y=2$.
(g) $R$ is the region bounded by $y=x$ and $y=\sqrt{x}$; about the line $x=2$.
9. Find the volume of the cone obtained by rotating the region under the segment joining $(0, h)$ and $(r, 0)$ about the $y$-axis.
10. The torus is the solid obtained by rotating the circle $(x-a)^{2}+y^{2}=b^{2}$ around the $y$-axis (assume that $a>b$ ). Show that it has volume $2 \pi^{2} a b^{2}$.
[Hint: Draw a picture, set up the problem and evaluate the integral by interpreting it as the area of a circle.]
