

**MA 114 Worksheet #15: Taylor and Maclaurin Series**

1. (a) Suppose that  $f(x)$  has a power series representation for  $|x| < R$ . What is the general formula for the Maclaurin series for  $f$ ?  
(b) Suppose that  $f(x)$  has a power series representation for  $|x - a| < R$ . What is the general formula for the Taylor series for  $f$  about  $a$ ?  
(c) Let  $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$ . Find the Maclaurin series for  $f$ .  
(d) Let  $f(x) = 1 + 2x + 3x^2 + 4x^3$ . Find the Taylor series for  $f(x)$  centered at  $x = 1$ .
2. Assume that each of the following functions has a power series expansion. Find the Maclaurin series for each. Be sure to provide the domain on which the expansion is valid.

(a)  $f(x) = \ln(1 + x)$

(b)  $f(x) = xe^{2x}$

3. Use a known Maclaurin series to obtain the Maclaurin series for the given function. Specify the radius of convergence for the series.

(a)  $f(x) = \frac{x^2}{1 - 3x}$

(d)  $f(x) = x^5 \sin(3x^2)$

(b)  $f(x) = e^x + e^{-x}$

(e)  $f(x) = \sin^2 x$ .

(c)  $f(x) = e^{-x^2}$

HINT:  $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$

4. Find the following Taylor expansions about  $x = a$  for each of the following functions and their associated radii of convergence.

(a)  $f(x) = e^{5x}$ ,  $a = 0$ .

(b)  $f(x) = \sin(\pi x)$ ,  $a = 1$ .

5. Differentiate the series

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

to find a Taylor series for  $\cos(x)$ .

6. Use Maclaurin series to find the following limit:  $\lim_{x \rightarrow 0} \frac{x - \tan^{-1}(x)}{x^3}$ .
7. Approximate the following integral using a 6th order polynomial for  $\cos(x)$ .

$$\int_0^1 x \cos(x^3) dx$$

8. Use power series multiplication to find the first three terms of the Maclaurin series for

$$f(x) = e^x \ln(1 - x).$$

## MA 114 Math Excel Worksheet # 15: Taylor Series & Taylor Polynomials

1. Using the Maclaurin series of  $\frac{1}{1-x}$ , find the Maclaurin series of  $\frac{1}{(1-x)^2}$
2. Find the first three terms of the Maclaurin series of  $f(x)$  and use it to calculate  $f^{(3)}(0)$ .
  - (a)  $f(x) = (x^2 - x)e^{x^2}$
  - (b)  $f(x) = \arctan(x^2 - x)$
3. Calculate  $\frac{\pi}{2} - \frac{\pi^3}{2^3 3!} + \frac{\pi^5}{2^5 5!} - \frac{\pi^7}{2^7 7!} + \cdots$ .