MA 114 Worksheet #11: Comparison and Limit Comparison Tests

- 1. (a) Explain the test for divergence. Why should you never use this test to prove that a series converges?
 - (b) State the comparison test for series. Explain the idea behind this test.
 - (c) Suppose that the sequences $\{x_n\}$ and $\{y_n\}$ satisfy $0 \le x_n \le y_n$ for all n and that $\sum_{n=1}^{\infty} y_n$ is convergent. What can you conclude? What can you conclude if instead $\sum_{n=1}^{\infty} y_n$ diverges?
 - (d) State the limit comparison test. Explain how you apply this test.
- 2. Use the appropriate test Divergence Test, Comparison Test or Limit Comparison Test to determine whether the infinite series is convergent or divergent.

(a)
$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2} + 1}$$

(b)
$$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n^2+2}}$$

(c)
$$\sum_{n=1}^{\infty} \frac{2^n}{2+5^n}$$

(d)
$$\sum_{n=0}^{\infty} \frac{4^n + 2}{3^n + 1}$$

(e)
$$\sum_{n=1}^{\infty} \left(\frac{10}{n}\right)^{10}$$

(f)
$$\sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}}$$

(g)
$$\sum_{n=1}^{\infty} \frac{n^2 + n + 1}{3n^2 + 14n + 7}$$

(h)
$$\sum_{n=0}^{\infty} \frac{1+2^n}{2+5^n}$$

(i)
$$\sum_{n=1}^{\infty} \frac{2}{n^2 + 5n + 2}$$

$$(j) \sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$$

$$(k) \sum_{n=0}^{\infty} \frac{n}{n^2 - \cos^2 n}$$

(l)
$$\sum_{n=1}^{\infty} \frac{n!}{n^4}$$

(m)
$$\sum_{n=0}^{\infty} \frac{n^2}{(n+1)!}$$

MathExcel Worksheet #11: Comparison and Limit Comparison Tests

Worksheet 11

1. For each of the following, determine which of the two is greater (for large n).

(a)
$$\frac{1}{n}$$
, $\frac{1}{n^2}$

(a)
$$n$$
, n^2
(b) $\frac{1}{e^n}$, $\frac{1}{10^n}$

(c)
$$\frac{1000}{n!}$$
, $\frac{2}{7^n}$

(d)
$$\frac{1}{\ln(7n)}$$
, $\frac{1}{(12n)^2}$

(e)
$$\frac{4}{\sqrt{n^5+8}}$$
, $\frac{1}{200n^2}$

(f)
$$\frac{1}{10^n}$$
, $\frac{1}{n^{10}}$

2. (Review) For each of the following geometric series, determine the exact value of the sum.

(a)
$$\sum_{n=1}^{\infty} 5^{-n}$$

(c)
$$\sum_{n=-1}^{\infty} 0.75^n$$

(b)
$$\sum_{n=2}^{\infty} \frac{3}{(-2)^n}$$

(d)
$$\sum_{n=0}^{\infty} \frac{2+4^n}{5^n}$$