

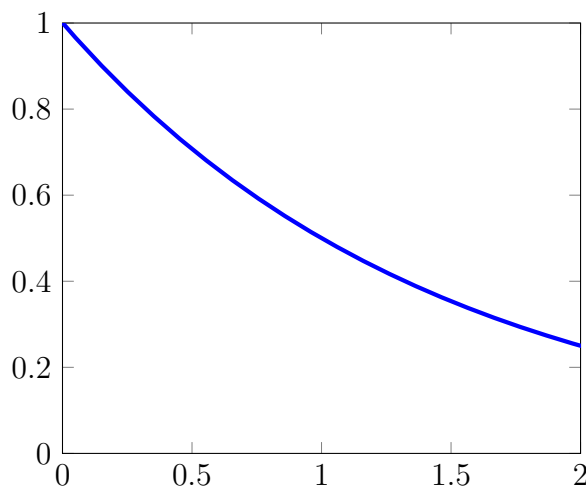
MA 114 Worksheet #05: Numerical Integration

- Write down the Midpoint rule and illustrate how it works with a sketch.
 - Write down the Trapezoid rule, illustrate how it works with a sketch, and write down the error bound associated with it.
 - How large should n be in the Midpoint rule so that you can approximate

$$\int_0^1 \sin x \, dx$$

with an error less than 10^{-7} ?

- Use the Midpoint rule to approximate the value of $\int_{-1}^1 e^{-x^2} \, dx$ with $n = 4$. Draw a sketch to determine if the approximation is an overestimate or an underestimate of the integral.
- The left, right, Trapezoidal, and Midpoint Rule approximations were used to estimate $\int_0^2 f(x) \, dx$, where f is the function whose graph is shown. The estimates were 0.7811, 0.8675, 0.8632, and 0.9540, and the same number of sub-intervals were used in each case.
 - Which rule produced which estimate?
 - Between which two approximations does the true value of $\int_0^2 f(x) \, dx$ lie?



- Draw the graph of $f(x) = \sin\left(\frac{1}{2}x^2\right)$ in the viewing rectangle $[0, 1]$ by $[0, 0.5]$ and let $I = \int_0^1 f(x) \, dx$.
 - Use the graph to decide whether L_2 , R_2 , M_2 , and T_2 underestimate or overestimate I .
 - For any value of n , list the numbers L_n , R_n , M_n , T_n , and I in increasing order.
 - Compute L_5 , R_5 , M_5 , and T_5 . From the graph, which do you think gives the best estimate of I ?

5. The velocity in meters per second for a particle traveling along the axis is given in the table below. Use the Midpoint rule and Trapezoid rule to approximate the total distance the particle traveled from $t = 0$ to $t = 6$.

t	$v(t)$
0	0.75
1	1.34
2	1.5
3	1.9
4	2.5
5	3.2
6	3.0

Math Excel Worksheet # 5: Numerical Integration

1. The function f is continuous on the closed interval $[2, 14]$ and has values as show in the table below. Using three subintervals, what is the approximation of $\int_2^{14} f(x)dx$ using both the Midpoint and Trapezoid rule?

x	2	6	10	14
f(x)	12	28	34	30

2. (a) State Simpson's rule for approximating $\int_a^b f(x) dx$ using $N = 4$ intervals of size Δx .
- (b) The identity $\int_1^2 \frac{1}{x} dx = \ln(2)$ gives us a way to compute $\ln(2)$ using Simpson's rule. How many intervals N would be required to use Simpson's rule in order to compute $\ln(2)$ with an error of no more than 5×10^{-5} ? Recall that the error estimate for Simpson's rule applied to $\int_a^b f(x) dx$ is $\text{Error}(S_N) \leq \frac{K_4(b-a)^5}{180N^4}$.