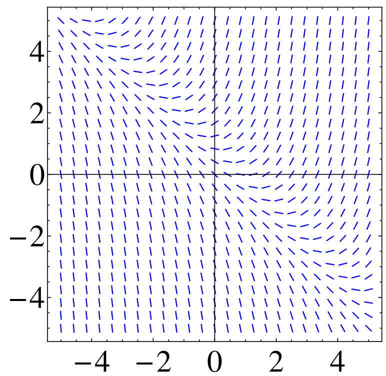
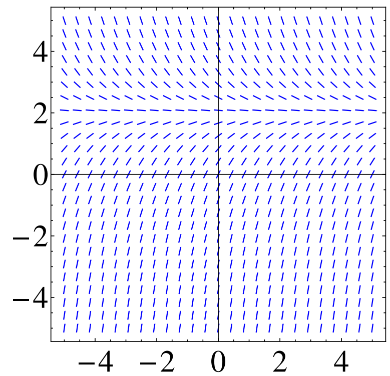
MA 114 Worksheet #28: Direction fields, Separable Differential Equations

1. Match the differential equation with its slope field. Give reasons for your answer.

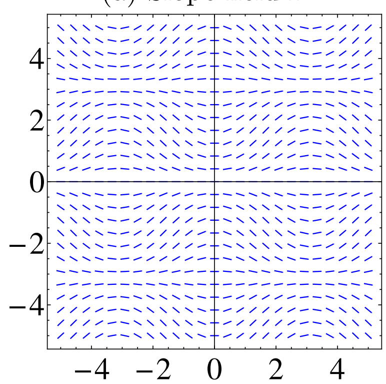
$$y' = 2 - y \qquad y' = x(2-y) \qquad y' = x + y - 1 \qquad y' = \sin(x)\sin(y)$$



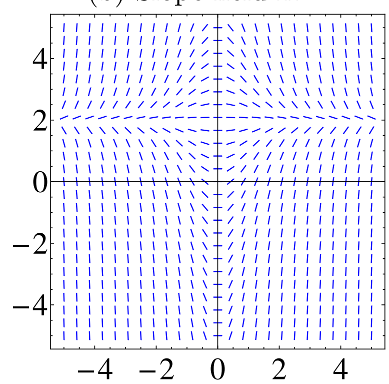
(a) Slope field I



(b) Slope field II



(c) Slope Field III



(d) Slope field IV

Figure 1: Slope fields for Problem 1

2. Use slope field labeled IV to sketch the graphs of the solutions that satisfy the given initial conditions.

$$y(0) = -1, \quad y(0) = 0, \quad y(0) = 1.$$

3. Sketch the slope field of the differential equation. Then use it to sketch a solution curve that passes through the given point.
   (a) $y' = y^2$, $(1, 1)$
   (b) $y' = y - 2x$, $(1, 0)$
   (c) $y' = xy - x^2$, $(0, 1)$

4. Consider the autonomous (depends only on y and its derivatives) differential equation $y' = y^2(3-y)(y+1)$. Without solving the differential equation, determine the value of $\lim_{t\to\infty} y(t)$, where the initial value is
   (a) $y(0) = 1$,
   (b) $y(0) = 4$,
   (c) $y(0) = -4$.

5. Use Euler's method with step size 0.5 to compute the approximate y-values, y_1, y_2, y_3, and y_4 of the solution of the initial-value problem $y' = y - 2x$, $y(1) = 0$.

6. Use separation of variables to find the general solutions to the following differential equations.
   (a) $y' + 4xy^2 = 0$
   (b) $\sqrt{1-x^2}y' = xy$
   (c) $(1+x^2)y' = x^3y$
   (d) $y' = 3y - y^2$

MA 114 MathExcel Worksheet # 28: Differential Equations, Direction Fields, Euler's Method

1. A tank contains 20 kg of salt dissolved in 5000 L of water. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 25 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. Formulate an initial value problem whose solution is the quantity of salt in the tank at time t.

2. Sketch the direction field for the differential equation $y' = x^2 + y^2 - 1$.

3. Use problem (2) to sketch the solution curve that passes through the origin.

4. Review: Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
   (a) $y = \cos(x)$, $y = 0$, $x = \frac{3\pi}{2}$, $x = \frac{5\pi}{2}$; about the y-axis
   (b) $y = x^3$, $y = x^2$; about $y = 1$
   (c) $y = x^3$, $y = 8$, $x = 0$; about $x = 2$

5. Review: The base of a solid is a circular disk with radius 3. Find the volume of the solid if the parallel cross-sections perpendicular to the base are isosceles right triangles with hypotenuse lying along the base.

6. Review: Find the centroid of the region bounded by $y = 4 - x^2$ and $y = x + 2$.

7. Review:

   (a) Find the area bounded by one leaf in the graph of polar function $r(\theta) = 2\sin(3\theta)$.

   (b) Suppose we cut the leaves in part (a) along the curve $r(\theta) = 1$. Find the area of one leaf tip.

   Hint: The integral bounds are not the same as in part (a)!

8. Review: Identify each conic section as a circle, ellipse, parabola, or hyperbola. Describe geometric features such as center, vertex, focus, and directrix wherever they are applicable.

   (a) $9y^2 - x + 3 = 0$

   (b) $y^2 - 6y + x^2 + 2x + 6 = 0$

   (c) $4y^2 - x^2 - 36 = 0$

   (d) $x^2 - 4x + 4y^2 = 0$