

## MA 114 Worksheet #18: Arc length and Surface area

1.
  - (a) Write down the formula for the arc length of a function  $f(x)$  over the interval  $[a, b]$  including the required conditions on  $f(x)$ .
  - (b) Write down the formula for the surface area of a solid of revolution generated by rotating a function  $f(x)$  over the interval  $[a, b]$  around the  $x$ -axis. Include the required conditions on  $f(x)$ .
  - (c) Write down the formula for the surface area of a solid of revolution generated by rotating a function  $f(x)$  over the interval  $[a, b]$  around the  $y$ -axis. Include the required conditions on  $f(x)$ .
2. Find an integral expression for the arc length of the following curves. Do **not** evaluate the integrals.
  - (a)  $f(x) = \sin(x)$  from  $x = 0$  to  $x = 2$ .
  - (b)  $f(x) = x^4$  from  $x = 2$  to  $x = 6$ .
  - (c)  $x^2 + y^2 = 1$
3. Find the arc length of the following curves.
  - (a)  $f(x) = x^{3/2}$  from  $x = 0$  to  $x = 2$ .
  - (b)  $f(x) = \ln(\cos(x))$  from  $x = 0$  to  $x = \pi/3$ .
  - (c)  $f(x) = e^x$  from  $x = 0$  to  $x = 1$ .
4. Set up a function  $s(t)$  that gives the arc length of the curve  $f(x) = 2x + 1$  from  $x = 0$  to  $x = t$ . Find  $s(4)$ .
5. Compute the surface areas of revolution about the  $x$ -axis over the given interval for the following functions.
  - (a)  $y = x$ ,  $[0, 4]$
  - (b)  $y = x^3$ ,  $[0, 2]$
  - (c)  $y = (4 - x^{2/3})^{3/2}$ ,  $[0, 8]$
  - (d)  $y = e^{-x}$ ,  $[0, 1]$
  - (e)  $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ ,  $[1, e]$
  - (f)  $y = \sin x$ ,  $[0, \pi]$
  - (g) Find the surface area of the torus obtained by rotating the circle  $x^2 + (y - b)^2 = r^2$  about the  $x$ -axis.
  - (h) Show that the surface area of a right circular cone of radius  $r$  and height  $h$  is  $\pi r \sqrt{r^2 + h^2}$ .  
Hint: Rotate a line  $y = mx$  about the  $x$ -axis for  $0 \leq x \leq h$ , where  $m$  is determined by the radius  $r$ .