MA 114 Worksheet #17: Volumes II

- 1. (a) Write a general integral to compute the volume of a solid obtained by rotating the region under y = f(x) over the interval [a, b] about the y-axis using the method of cylindrical shells.
 - (b) If you use the disk method to compute the same volume, are you integrating with respect to x or y? Why?
- 2. Sketch the enclosed region and use the Shell Method to calculate the volume of rotation about the y-axis.
 - (a) y = 3x 2, y = 6 x, x = 0
 - (b) $y = x^2$, $y = 8 x^2$, x = 0, for $x \ge 0$
 - (c) $y = 8 x^3$, y = 8 4x, for $x \ge 0$
- 3. For each of the following, use disks or washers to find the an integral expression for the volume of the region. Evaluate the integrals for parts (a) and (d).
 - (a) R is region bounded by $y = 1 x^2$ and y = 0; about the x-axis.
 - (b) R is region bounded by $y = \frac{1}{x}$, x = 1, x = 2, and y = 0; about the x-axis.
 - (c) R is region bounded by $x = 2\sqrt{y}$, x = 0, and y = 9; about the y-axis.
 - (d) R is region bounded by $y = 1 x^2$ and y = 0; about the line y = -1.
 - (e) Between the regions in part (a) and part (d), which volume is bigger? Why?
 - (f) R is region bounded by $y = e^{-x}$, y = 1, and x = 2; about the line y = 2.
 - (g) R is region bounded by y = x and $y = \sqrt{x}$; about the line x = 2.
- 4. A soda glass has the shape of the surface generated by revolving the graph of $y = 6x^2$ for $0 \le x \le 1$ about the y-axis. Soda is extracted from the glass through a straw at the rate of 1/2 cubic inch per second. How fast is the soda level in the glass dropping when the level is 2 inches? (Answer should be implicitly in units of inches per second.)
- 5. The torus is the solid obtained by rotating the circle $(x-a)^2 + y^2 = b^2$ around the y-axis (assume that a > b). Show that it has volume $2\pi^2 a b^2$. [Hint: Draw a picture, set up the problem and evaluate the integral by interpreting it as the area of a circle.]