Spring 2017

## MA 114 Worksheet #11: Alternating Series and Absolute/Conditional Convergence

- 1. (a) Let  $a_n = \frac{n}{3n+1}$ . Does  $\{a_n\}$  converge? Does  $\sum_{n=1}^{\infty} a_n$  converge?
  - (b) Give an example of a divergent series  $\sum_{n=1}^{\infty} a_n$  where  $\lim_{n \to \infty} a_n = 0$ .
  - (c) Does there exist a convergent series  $\sum_{n=1}^{\infty} a_n$  which satisfies  $\lim_{n \to \infty} a_n \neq 0$ ? Explain.
  - (d) When does a series converge absolutely? When does a series converge conditionally?
  - (e) State the alternating series test.
  - (f) Prove that the alternating harmonic series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$  converges.
  - (g) State the Alternating Series Estimation Theorem.
- 2. Test the following series for convergence or divergence.

(a) 
$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2n}$$
 (d)  $\sum_{n=1}^{\infty} \frac{3^n}{4^n+5^n}$   
(b)  $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$  (e)  $\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$   
(c)  $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{2/3}}$  (f)  $\sum_{n=1}^{\infty} \left(\frac{-5}{18}\right)$ 

3. Use the Alternating Series Estimation Theorem to estimate the sum correct to four decimal places.

(a) 
$$\sum_{n=1}^{\infty} \frac{(-0.8)^n}{n!}$$
  
(b)  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n}$ 

4. Approximate the sum of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(2n)!}$  correct to four decimal places; *i. e.* so that |error| < 0.00005.