

## MA 114 Worksheet #06: Sequences

1. (a) Give the precise definition of a **sequence**.  
(b) What does it mean to say that  $\lim_{x \rightarrow a} f(x) = L$  when  $a = \infty$ ? Does this differ from  $\lim_{n \rightarrow \infty} f(n) = L$ ? Why or why not?  
(c) What does it mean for a sequence to converge? Explain your idea, not just the definition in the book.  
(d) Sequences can diverge in different ways. Describe two distinct ways that a sequence can diverge.  
(e) Give two examples of sequences which converge to 0 and two examples of sequences which converges to a given number  $L \neq 0$ .
2. Write the first four terms of the sequences with the following general terms:  
(a)  $\frac{n!}{2^n}$   
(b)  $\frac{n}{n+1}$   
(c)  $(-1)^{n+1}$   
(d)  $\{a_n\}_{n=1}^{\infty}$  where  $a_n = \frac{3}{n}$ .  
(e)  $\{a_n\}_{n=1}^{\infty}$  where  $a_n = 2^{-n} + 2$ .  
(f)  $\{b_k\}_{k=1}^{\infty}$  where  $b_k = \frac{(-1)^k}{k^2}$ .
3. Find a formula for the  $n$ th term of each sequence.  
(a)  $\left\{ \frac{1}{1}, -\frac{1}{8}, \frac{1}{27}, -\frac{1}{64}, \dots \right\}$   
(b)  $\left\{ 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots \right\}$   
(c)  $\{1, 0, 1, 0, 1, 0, \dots\}$   
(d)  $\left\{ -\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \dots \right\}$
4. Suppose that a sequence  $\{a_n\}$  is bounded above and below. Does it converge? If not, find a counterexample.
5. The limit laws for sequences are the same as the limit laws for functions. Suppose you have sequences  $\{a_n\}$ ,  $\{b_n\}$  and  $\{c_n\}$  with  $\lim_{n \rightarrow \infty} a_n = 15$ ,  $\lim_{n \rightarrow \infty} b_n = 0$  and  $\lim_{n \rightarrow \infty} c_n = 1$ . Use the limit laws of sequences to answer the following questions.  
(a) Does the sequence  $\left\{ \frac{a_n \cdot c_n}{b_n + 1} \right\}_{n=1}^{\infty}$  converge? If so, what is its limit?  
(b) Does the sequence  $\left\{ \frac{a_n + 3 \cdot c_n}{2 \cdot b_n + 2} \right\}_{n=1}^{\infty}$  converge? If so, what is its limit?