

MA 114 Worksheet #05: Simpson's Rule and Improper Integrals

- Write down Simpson's rule and illustrate how it works with a sketch.
 - How large should n be in the Simpson's rule so that you can approximate

$$\int_0^1 \sin x \, dx$$

with an error less than 10^{-7} ?

- Approximate the integral $\int_1^2 \frac{1}{x} \, dx$ using Simpson's rule. Choose n so that your error is certain to be less than 10^{-3} . Compute the exact value of the integral and compare to your approximation.
- Simpson's Rule turns out to exactly integrate polynomials of degree three or less. Show that Simpson's rule gives the *exact* value of $\int_0^h p(x) \, dx$ where $h > 0$ and $p(x) = ax^3 + bx^2 + cx + d$. [Hint: First compute the exact value of the integral by direct integration. Then apply Simpson's rule with $n = 2$ and observe that the approximation and the exact value are the same.]
- For each of the following, determine if the integral is proper or improper. If it is improper, explain why. Do **not** evaluate any of the integrals.

(a) $\int_0^2 \frac{x}{x^2 - 5x + 6} \, dx$

(d) $\int_{-\infty}^{\infty} \frac{\sin x}{1 + x^2} \, dx$

(b) $\int_1^2 \frac{1}{2x - 1} \, dx$

(e) $\int_0^{\pi/2} \sec x \, dx$

(c) $\int_1^2 \ln(x - 1) \, dx$

- For the integrals below, determine if the integral is convergent or divergent. Evaluate the convergent integrals.

(a) $\int_{-\infty}^0 \frac{1}{2x - 1} \, dx$

(c) $\int_0^2 \frac{x - 3}{2x - 3} \, dx$

(b) $\int_{-\infty}^{\infty} x e^{-x^2} \, dx$

(d) $\int_0^{\infty} \sin \theta \, d\theta$

- Consider the improper integral

$$\int_1^{\infty} \frac{1}{x^p} \, dx.$$

Integrate using the generic parameter p to prove the integral converges for $p > 1$ and diverges for $p \leq 1$. You will have to distinguish between the cases when $p = 1$ and $p \neq 1$ when you integrate.

7. Use the Comparison Theorem to determine whether the following integrals are convergent or divergent.

(a) $\int_1^{\infty} \frac{2 + e^{-x}}{x} dx$

(b) $\int_1^{\infty} \frac{x + 1}{\sqrt{x^6 + x}} dx$

8. Explain why the following computation is wrong and determine the correct answer. (Try sketching or graphing the integrand to see where the problem lies.)

$$\begin{aligned} \int_2^{10} \frac{1}{2x-8} dx &= \frac{1}{2} \int_{-4}^{12} \frac{1}{u} du \\ &= \frac{1}{2} \ln |x| \Big|_{-4}^{12} \\ &= \frac{1}{2} (\ln 12 - \ln 4) \end{aligned}$$

where we used the substitution

$$\begin{cases} u(x) = 2x - 8 \\ u(2) = -4 & u(10) = 12 \\ \frac{du}{dx} = 2 \end{cases}$$

9. A manufacturer of light bulbs wants to produce bulbs that last about 700 hours but, of course, some bulbs burn out faster than others. Let $F(t)$ be the fraction of the company's bulbs that burn out before t hours, so $F(t)$ always lies between 0 and 1.
- (a) Make a rough sketch of what you think the graph of F might look like.
- (b) What is the meaning of the derivative $r(t) = F'(t)$?
- (c) What is the value of $\int_0^{\infty} r(t) dt$? Why?