# Math 2551 Worksheet Answers - Review from Single-Variable Calculus and Linear Algebra

1. $2\mathbf{v} = \langle 2, 4, 2\rangle$, $\mathbf{v} - \mathbf{w} = \langle 2, 2, -1\rangle$, $\mathbf{v} \cdot \mathbf{w} = 1$.
2. $a + 15b + 7c$
3. $\mathbf{u} \cdot \mathbf{v} = 0$ (because the vectors are orthogonal)
4. (a) $\mathbf{v} \cdot \mathbf{u} = -25$
   (b) $\cos(\theta) = \frac{-25}{25} = -1$
   (c) $(3\mathbf{v}) \cdot (2\mathbf{u}) = 3(2)(-25) = -150$.
5. (a) 16/15
   (b) $\frac{\pi}{2}$
   (c) $\ln(4) - \frac{3}{4}$
6. $f_{max} = 2e^4$, since the function has no critical points and is increasing on this interval.
7. Area $= \frac{9}{2}$.

# Math 2551 Worksheet Answers: $\mathbb{R}^3$

1. $x = 0, y = 3\sqrt{3}/2, z = 3/2$.
2. The first car is faster, at approx 43.28 km/h.
3. $(x-1)^2 + (y+2)^2 + (z-3)^2 = 16$.

# Math 2551 Worksheet 2 Answers: $\mathbb{R}^3$ and Cross Products

1. Let $P = (1, -1, 2)$, $Q = (2, 0, -1)$, and $R = (0, 2, 1)$.
   (a) $2\sqrt{6}$
   (b) $\frac{1}{\sqrt{6}}\langle 2, 1, 1\rangle$
2. (a) True
   (b) True
   (c) True
3. (a) Makes sense, scalar
   (b) Does not make sense
   (c) Makes sense, vector
   (d) Does not make sense
4. Max value is $\sqrt{13}$, given by $\mathbf{v} = \mathbf{u}/|\mathbf{u}|$ and min value is $-\sqrt{13}$, given by $\mathbf{v} = -\mathbf{u}/|\mathbf{u}|$.

# Math 2551 Worksheet Answers: Lines and Planes

1. (a) $\mathbf{r}(t) = \langle 1 - 2t, 2 - 2t, -1 + 2t \rangle$
   (b) $\mathbf{r}(t) = \langle t, -7 + 2t, 2t \rangle$
   (c) $\mathbf{r}(t) = \langle 1 + 14t, 2t, 15t \rangle$
2. vector: $3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$.
   plane: $3(x - 2) - 3(y - 1) + 3(z + 1) = 0$
3. many correct solutions; pick two distinct points off of the line $PQ$ which are not collinear with the given points. Two such planes are $-2(x - 1) + 3(y + 1) + 5(z - 1) = 0$ and $-3(x - 1) + 3(y + 1) + 3(z - 1) = 0$.
4. $(2, 7, 3)$
5. Parallel: $\langle A_1, B_1, C_1 \rangle = \lambda \langle A_2, B_2, C_2 \rangle$ for some $\lambda \neq 0$
   Perpendicular: $\langle A_1, B_1, C_1 \rangle \cdot \langle A_2, B_2, C_2 \rangle = 0$
6. (a) distance $= \dfrac{\vec{QP} \cdot \mathbf{n}}{\mathbf{n}}$ with $Q = (1, 0, 1)$ (any point on the plane works). So the distance is $\dfrac{\langle 0, 2, 2 \rangle \cdot \langle 2, -1, 3 \rangle}{|\langle 2, -1, 3 \rangle|} = \dfrac{4}{\sqrt{14}}$
   (b) distance is $|\vec{QP}| \sin(\theta) = \dfrac{|\vec{QP} \times \mathbf{v}|}{|\mathbf{v}|}$ where $Q = (1, 1, 1), \mathbf{v} = \langle 2, 3, -1 \rangle$ (any point on the line and any direction vector works). So the distance is $\dfrac{\sqrt{69}}{\sqrt{14}}$.

# Math 2551 Worksheet Answers: Quadric Surfaces

1. (a) elliptical cylinder, oriented along the $y$-axis, cross-sections are ellipses in the $y = k$ planes or vertical/horizontal lines in the $x = k$ and $z = k$ planes.

   (b) ellipsoid, centered at the origin, wider in the $z$ and $y$ directions than the $x$ direction, cross-sections are circles in the $x = k$ planes (if $k < 1$), ellipses in the $z = k$ and $y = k$ planes ($k < 3$)

   (c) circular cone, oriented along the $y$-axis, cross sections are circles in the $y = k$ planes and lines in the $x = k$ and $z = k$ planes

   (d) elliptical paraboloid, oriented in the positive $z$ direction, shifted up 4 units, cross-sections are circles in the $z = k$ planes for $k \geq 4$ and parabolas in the $x = k$ or $y = k$ planes

   (e) sphere, centered at $(0, 0, 0)$, radius 4, cross-sections are circles in $x = k, y = k$, and $z = k$ planes for $0 \leq k \leq 4$.

# Math 2551 Worksheet Answers: Curves in Space and Their Tangents

1. This curve's graph is a spiral, narrowing to a point at the origin when $t = 0$ and widening outward around the $z$-axis for larger/smaller $t$.

2. $\mathbf{r}(t) = \langle 3\cos(t), 3\sin(t), 2 - 3\sin(t)\rangle, 0 \leq t \leq 2\pi$.

3. All three functions describe part of the same set of points in $\mathbb{R}^3$, which lie above the line $y = x$ in the $xy$-plane and form a parabola in the plane $x = y$. $\mathbf{f}$ traces out all of the points on this parabola, $\mathbf{g}$ only those in the first octant, and $\mathbf{h}$ only those which lie above the square $[-1, 1] \times [-1, 1]$.

4. Many possible answers; depending on the domain and functions involved. If the domain is bounded, e.g. $[a, b]$, then letting $s = b + (a - b)t$ and taking $\mathbf{r}(s)$ as the new parametric equations works. If the domain is $(-\infty, \infty)$, we can just let $s = -t$.

5. (a) $y = \frac{2}{9}x^2$ for $x > 0$

   (b) $\mathbf{v}(\ln(3)) = 3\mathbf{i} + 4\mathbf{j}$
   $\mathbf{a}(\ln(3)) = 3\mathbf{i} + 8\mathbf{j}$.

6. $x(s) = s, y(s) = \frac{s}{3}, z(s) = s$

7. $(1, 1, 1)$ (where the first parameter is 1 and the second is 0). The angle is $\arccos(3/\sqrt{14})$. The bees would not collide, since the first bee reaches the point at $t = 1$ and the second bee at $t = 0$.

8. $-\frac{1}{\sqrt{2}}(x - \frac{1}{\sqrt{2}}) + \frac{1}{\sqrt{2}}(y - \frac{1}{\sqrt{2}}) + 6(z - 0) = 0$

   OR $x - y - 6\sqrt{2}z = 0$

# Math 2551 Worksheet 5 Answers: Calculus of Vector-Valued Functions

1. $\mathbf{r}(t) = \langle -\frac{1}{2}t^2 + 5t + 10, -\frac{1}{2}t^2 + 10, -\frac{1}{2}t^2 + 10\rangle, \quad t \geq 0.$

2. A baseball is hit when it is 2.5 ft above the ground. It leaves the bat with an initial velocity of 140 ft/sec at a launch angle of 30°. At the instant the ball is hit, an instantaneous gust of wind blows against the ball, adding a component of $-14\hat{i}$ (ft/sec) to the ball's initial velocity. A 15 ft high fence lies 400 ft from the home plate in the direction of the flight. (Note that gravity, g = 32 ft/sec$^2$)

   (a) Sorry, no sketch. :)

   (b) $\vec{r}(t) = (140\cos 30^\circ - 14)t\hat{i} + (2.5 + (140\sin 30^\circ)t - 16t^2)\hat{j} = (70\sqrt{3} - 14)t\hat{i} + (2.5 + 70t - 16t^2)\hat{j}$.

   (c) $y_{\max} = \frac{(140\sin 30^\circ)^2}{64} + 2.5 = \frac{70^2}{64} + 2.5 = 79.0625$ ft., which is reached at $t = \frac{140\sin 30^\circ}{32} = \frac{70}{32} = 2.1875$ s.

   (d) For the time, solve $y = 2.5 + 70t - 16t^2 = 0$ for $t$. Using quadratic formula, we have $t = 4.41$s. Then, the range at $t = 4.41$ is $x(4.41) = (140\cos 30^\circ - 14)(4.41) = 472.94$ ft.

   (e) For the time, solve $y = 2.5 + 70t - 16t^2 = 20$ for $t$. Using quadratic formula, we have $t = 0.27,\ 4.11$ seconds. Then, the range at those times are $x(0.27) = 29$ ft and $x(4.11) = 441$ ft.

   (f) Yes, according to part (d), the ball is still 20 feet above the ground when it is 441 feet from home plate.

# Math 2551 Worksheet Answers: Arc Length

1. $\mathbf{T}(t) = \frac{1}{13}\langle 12\cos(2t), -12\sin(2t), 5\rangle$
   length: $13\pi$
2. $(0, 5, 24\pi)$
3. $\int_0^1 \sqrt{(\frac{2}{t+1})^2 + (2e^{2t}+1)^2 + (2\sin(t)\cos(t))^2}\, dt$
4. $\sqrt{6}$

# Math 2551 Worksheet Answers: Curvature and Normals

1. $\mathbf{T}(t) = \frac{1}{1+e^{2t}}\langle\sqrt{2}e^t, e^{2t}, -1\rangle$

   $\mathbf{N}(t) = \frac{1}{1+e^{2t}}\langle 1-e^{2t}, \sqrt{2}e^t, \sqrt{2}e^t\rangle$

   $\kappa(t) = \frac{\sqrt{2}e^{2t}}{(e^{2t}+1)^2}$

2. $\mathbf{T}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$

   $\mathbf{N}(t) = -\sin(t)\mathbf{i} + \cos(t)\mathbf{j}$

   $\kappa(t) = \frac{1}{t}$

3. $\mathbf{T} = \langle\frac{1}{1+t^2}, \frac{t^2}{1+t^2}\rangle$

   $\mathbf{N} = \langle\frac{-t}{\sqrt{2t^2}}, \frac{t}{\sqrt{2t^2}}\rangle$

   The normal vector does not exist when $t = 0$; as $t$ passes from negative to positive values the normal vector changes which side of the curve it is on.

4. The point of greatest curvature occurs at $x = 0$. Using the parameterization $\mathbf{r}(t) = \langle t, t^2\rangle$ gives $\kappa(t) = \frac{2}{(1+t^2)^{3/2}}$, which is maximized when $t = 0$.

# Math 2551 Worksheet Answers: Multivariable Functions

1. Find and sketch the domain for each function.
   (a) $\{(x, y) \mid x - y \geq 1\}$
   (b) $\{(x, y) \mid x \geq 4, |y| \geq 1\} \cup \{(x, y) \mid x < 4, |y| < 1\}$
   (c) $\{(x, y) \mid 4x^2 - 1 \leq y \leq 4x^2 + 1\}$
   (d) All of $\mathbb{R}^2$ except the circle $x^2 + y^2 = 4$
   (e) All of the disk $x^2 + y^2 < 4$ except the circle $x^2 + y^2 = 3$.

2. (a) a collection of concentric ellipses
   (b) a collection of unequally spaced concentric circles
   (c) a collection of unequally spaced parallel lines
   (d) a collection of equally spaced parallel lines
   (e) a collection of unequally spaced concentric circles
   (f) a collection of equally spaced concentric circles
   (g) two straight lines and a collection of hyperbolas

3. The plane $4x + y + 3 = 0$, except for those points with $x + y = -1$.

4. The sphere $3 = x^2 + y^2 + z^2$

5. When $x = 0$, the cross-section is the parabola $z = y^2$.
   When $y = 0$, the cross-section is the parabola $z = x^2$.
   When $x = y$, the cross-section is the line $z = 0$.
   The level curves are pairs of parallel lines $y = x \pm \sqrt{k}$.

# Math 2551 Worksheet Answers: Limits and Continuity

1. $\frac{1}{36}$
2. $\frac{1}{12}$
3. $\frac{1}{4}$
4. Does not exist.
5. $f$ is continuous on its entire domain: all $(x, y)$ such that neither $x = 0$ nor $y = 0$.
6. $f$ is continuous on its entire domain: all $(x, y, z)$ except the sphere $x^2 + y^2 + z^2 = e$.

# Math 2551 Worksheet 8 Answers - Review for Exam 1

1. $\int_0^1 \sqrt{1+e^{2t}}\, dt$
2. $\ell(s) = \langle 0, -s, -s \rangle$
3. $\mathbf{T}(t) = \langle -\sin(t), \cos(t) \rangle$

   $\mathbf{N}(t) = \langle -\cos(t), -\sin(t) \rangle$

   Into
4. $\kappa = \dfrac{a}{1+a^2}$
5. $-3(x-3) + (y-2) - 2z = 0$

   Intersection point is $(0, 3, -2)$, when $t = 1$.
6. (a) $\{(x, y) \mid |x| \leq 3, |y| \geq 2\}$

   (b) $1 \leq x^2 + y^2 \leq 3$

   (c) $\dfrac{x^2}{16} + \dfrac{y^2}{4} \leq 1$
7. $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + \frac{2}{3}(t^{3/2} - 1)\mathbf{k}$
8. No, because the limit of $f$ as $(x, y) \to (0, 0)$ does not exist.

# Math 2551 Worksheet Answers: Partial Derivatives

1. (a) $f_x(-2,-1) \approx 0.75$
   (b) $f_y(-2,-1) \approx 1.5$
   (c) There are several possible points (these are places where the tangent to a contour is horizontal): $(0,-.5), (-.5,-1.25)$, etc.
   (d) Again, there are many possible points; any point on the 4, 5, 6 contours in quadrant IV will work.

2. (a) $C_T$: (meters/second)/ degree Celsius - this gives the change in speed for each one degree C of temperature increase. $C_S$ (meters/second)/(grams/liter) - this gives the change in speed for each one gram/liter increase in salinity $C_D$: (meters/second)/meter - this gives the change in speed for each one meter increase in depth below the surface
   (b) $C_T = 4.5 - 0.1T + 0.0009T^2 - 0.01(S-35)$ $C_S = 1.5 - 0.01T$ $C_D = 0.015$
   (c) At $(T,S,D) = (10,35,100)$, we have $C_T = 3.59, C_S = 0.5, C_D = 0.015$. This tells us that if we increase the temperature, salinity, or depth from these conditions the speed of sound will increase as well.

3. $f_{xx} = e^x, f_{xy} = f_{yx} = \dfrac{1}{y}, f_{yy} = -\dfrac{x}{y^2}$

4. $f_x = \sin(yz), f_y = xz\cos(yz), f_z = xy\cos(yz), f_{xzz} = -y^2\sin(yz)$

5. Note this does not have a definitive right answer - some differences may arise and that's good! Discuss!

   (a) First $y$ since $\partial^3 f/\partial y^3 = 0$ and the $y$-partial derivatives are easier
   (b) First $y$, since $\partial^3 f/\partial y^3 = 0$
   (c) First $y$, since $\partial^2 f/\partial y^2 = 0$
   (d) First $x$, since $\partial^2 f/\partial x^2 = 0$ and the $x$-partial derivatives are easier.

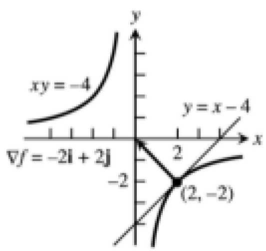
   A common theme is to work with the variable with lower powers/simpler expressions first when taking mixed partials.

# Math 2551 Worksheet Answers: Chain Rule

1. $\dfrac{dz}{dt}(t_0) = 1$
2. $t = -4/3$
3. $\dfrac{\partial w}{\partial r}(2, \pi/2) = 2\pi$ $\qquad$ $\dfrac{\partial w}{\partial \theta}(2, \pi/2) = -2\pi$
4. $\dfrac{dw}{dt}(2) = -10$. $\dfrac{dw}{dt}(1)$ cannot be computed from the given information because we do not know the values of $g_x$ or $g_y$ at $(x(1), y(1)) = (1, 3)$. We do not use the values of $g(1, 0), g(-1, 2), g_x(-1, 2), g_y(-1, 2)$.

# Math 2551 Worksheet Answers:Gradient and Directional Derivatives

1. (a) Negative
   (b) Negative
   (c) Approximately zero
   (d) Positive
   (e) Positive
2. Tangent line: $-2(x-2)+2(y+2)=0$




3. $D_{\mathbf{u}}g(1,-1) = \dfrac{21}{13}$
4. (a) Ascend at a rate of 0.8 vertical meters per horizontal meter
   (b) Descend at a rate of $\sqrt{2}/10$ vertical meters per horizontal meter
   (c) $\langle -0.6, -0.8\rangle$ is the direction of largest slope with rate of ascent 1 vertical meter per horizontal meter.
5. (a) $\langle -1/\sqrt{2}, 1/\sqrt{2}\rangle$
   (b) $2\sqrt{2}$
   (c) $\langle 1/\sqrt{2}, -1/\sqrt{2}\rangle$
   (d) $\langle 1/\sqrt{2}, 1/\sqrt{2}\rangle$

# Math 2551 Worksheet Answers: Linearization and Tangent Planes, Optimization

1. $L(x, y) = e^3 - e^3(x - 1) + 2e^3(y - 2)$
2. $L(x, y, z) = z$
3. $f(2.95, 7.1) = 4 - 1/40$
4. No, there are two different tangent planes. No, the function is not differentiable.
5. $z = 1 - 1/2(x - 1) + 1/2(y - 2)$

# Math 2551 Worksheet Answers: Optimization I

1. Saddle point at $(0,0)$ and local minimum at $(0,-2)$.
2. Saddle point at $(0,0)$ and local maximum at $(-1,-1)$.
3. Yes, this must be a saddle point because $f_{xx}(a,b)f_{yy}(a,b) < 0$ so $\det(Hf) = f_{xx}(a,b)f_{yy}(a,b) - f_{xy}^2(a,b) < 0$.
4. (a) Minimum is 0 at $(0,0)$ since $f(x,y) > 0$ for all other $(x,y)$.
   (b) Maximum is 1 at $(0,0)$ since $f(x,y) < 1$ for all other $(x,y)$.
   (c) Neither since $f(x,y) < 0$ for $x < 0$ and $f(x,y) > 0$ for $x > 0$.
   (d) Neither since $f(x,y) < 0$ for $x < 0$ and $f(x,y) > 0$ for $x > 0$.
   (e) Neither since $f(x,y) < 0$ for $x < 0$ and $y > 0$, but $f(x,y) > 0$ for $x > 0$ and $y > 0$.
   (f) Minimum is 0 at $(0,0)$ since $f(x,y) > 0$ for all other $(x,y)$.

# Math 2551 Worksheet Answers: Optimization II

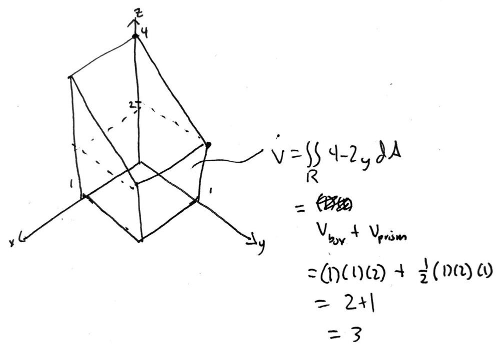
1. The absolute maximum is 17, achieved at $(0, 4)$ and $(4, 4)$, and the absolute minimum is 1, achieved at $(0, 0)$.

2. The dimensions are $3 \times 3 \times 3$ and the surface area is 54.

# Math 2551 Worksheet Answers: Lagrange Multipliers

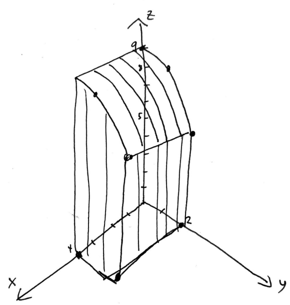
1. $V(2,2,1) = 4$ cubic units
2. The extreme values are 1 and 2.
3. The extreme values are 1 and $\sqrt{3}$.
4. The closest point is $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and the farthest point is $(-1, -1, 2)$.
5. $\frac{8r^3}{3\sqrt{3}}$

# Math 2551 Worksheet Answers: Double Integrals on Rectangles




1.

= 3




2.

3. $9\ln(2)$

4. $160/3$ cubic units.

5. $e^3 - 4$

6. The integrals evaluate to $\pi/4$ and $-\pi/4$ respectively. This does not violate Fubini's theorem because this function is not continuous on $[0, 1] \times [0, 1]$ (it has an asymptote at $(0, 0)$)

# Math 2551 Worksheet Answers: Double Integrals on General Regions

1. $2 + \frac{1}{2}\pi^2$
2. (a) Negative
   (b) Positive
   (c) Zero
   (d) Zero
   (e) Zero
3. (a) $\int_0^{\ln(3)} \int_{e^{-x}}^1 \, dy \, dx$ and $\int_{1/3}^1 \int_{-\ln(y)}^{\ln(3)} \, dx \, dy$
   (b) $\int_{-1}^2 \int_{x^2}^{x+2} \, dy \, dx$ and $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} \, dx \, dy + \int_1^4 \int_{y-2}^{\sqrt{y}} \, dx \, dy$
4. $\int_0^{\sqrt{\pi}} \int_0^x \cos(x^2) \, dy \, dx = 0$
5. $\dfrac{2}{15}$
6. 16/3
7. 32/3

# Math 2551 Worksheet 16 Answers: Applications, Polar Double Integrals

1. Answers will vary a bit through the estimation process

   4 subdivisions: $31.75 \leq T_{avg} \leq 52.5$
   16 subdivisions: $33.18 \leq T_{avg} \leq 50.06$
   25 subdivisions: $36.32 \leq T_{avg} \leq 49.8$

   Colorado is a rectangle, which makes it easy to subdivide. Wyoming would also work well.

2. On square: $f_{avg} = \frac{1}{1} \cdot \frac{1}{4} = \frac{1}{4}$

   On quarter circle: $f_{avg} = \frac{1}{\pi/4} \cdot \frac{1}{8} = \frac{1}{2\pi}$

3. 50 people

# Math 2551 Worksheet Answers: Polar Double Integrals

1. $5\pi - 26$
2. $\frac{\pi}{2}(1 - e^{-4})$.
3. $\frac{3\pi}{2} - 4$.
4. $256\pi$
5. (a) $I^2 = \lim_{R\to\infty} \int_0^2 \pi \int_0^R e^{-r^2} r \, dr \, d\theta$

   (b) $I^2 = \frac{\pi}{4}$, so $I = \frac{\sqrt{\pi}}{2}$

# Math 2551 Worksheet Answers: Exam 2 Review

1. All are true except b).
2. -1
3. $\pm 1$
4. Saddle at $(0, 0)$ with $f(0, 0) = 0$, local min at $(0, 2)$ of $-4$, local max at $(-2, 0)$ of 4, saddle at $(-2, 2)$ with $f(-2, 2) = 0$
5. $(-1/2, 1/2, 1/2)$ and $(0, 1, 0)$
6. sin(4)
7. No, this is less than $f(x, y)$ at all points, so it cannot possibly be the average value.
8. $1800\pi$ cubic feet

# Math 2551 Worksheet Answers: Triple Integrals

1. $\frac{-155}{2}$, region of integration the rectangular prism $[0,2]\times[-1,4]\times[0,1]$ or $0 \leq x \leq 2, -1 \leq y \leq 4, 0 \leq z \leq 1$.

2. $\frac{1}{24}$

3. $-\frac{1}{3}$

4. (a) $\int_0^1 \int_0^1 \int_{-1}^{-\sqrt{z}} dy\ dz\ dx$

   (b) $\int_0^1 \int_0^1 \int_{-1}^{-\sqrt{z}} dy\ dx\ dz$

   (c) $\int_0^1 \int_{-1}^{-\sqrt{z}} \int_0^1 dx\ dy\ dz$

   (d) $\int_{-1}^0 \int_0^{y^2} \int_0^1 dx\ dz\ dy$

   (e) $\int_{-1}^0 \int_0^1 \int_0^{y^2} dz\ dx\ dy$

5. $\int_0^2 \int_{-\sqrt{1-(y-1)^2}}^{\sqrt{1-(y-1)^2}} \int_{x^2+y^2}^{2y} dz\ dx\ dy$

   $\int_{-1}^1 \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \int_{x^2+y^2}^{2y} dz\ dy\ dx.$

# Math 2551 Worksheet Answers: Mass and Moments

1. $(\bar{x}, \bar{y}) = \left(0, \frac{9}{14}\right)$ and $I_y = \frac{16}{35}$
2. $(\bar{x}, \bar{y}) = \left(-\frac{1}{2}, 0\right)$

# Math 2551 Worksheet Answers: Triple Integrals in Cylindrical & Spherical Coordinates

1. $\dfrac{2}{5}$.
2. (a) $3\pi$.
   (b) $\bar{x} = 0$.
3. $\dfrac{12\sqrt{3}-4}{3}\pi$.
4. $\dfrac{a^3\pi}{18}$.
5. $\dfrac{8\sqrt{2}\pi}{3}$
6. $32\pi$

# Math 2551 Worksheet Answers: Change of Variables

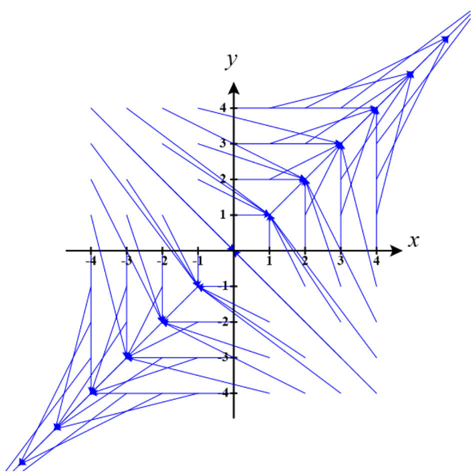
1. $\sin^2(\theta) - \cos^2(\theta) = -\cos(2\theta)$
2. The elliptical region $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \leq 1$.
3. $x = \sqrt{\dfrac{u}{v}}, y = \sqrt{uv}$ maps $[1,3] \times [1,3]$ onto $R$
4. $\displaystyle\int_0^1 \int_{3v}^{3v-15} -2v\frac{1}{5}\, du\, dv$
5. $\ln(9)$
6. $e + \dfrac{1}{e}$

# Math 2551 Worksheet Answers: Scalar Line Integrals

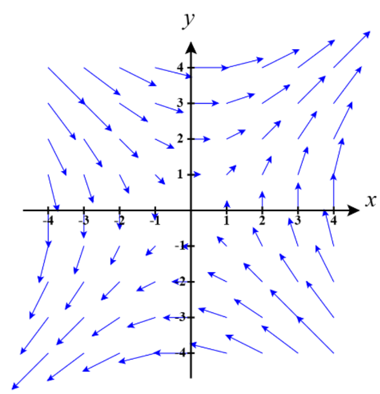
1. There are many possible correct answers! Here are some.
   (a) $\mathbf{r}(t) = \langle 3t, -2t+1, 4t-2 \rangle, \quad 0 \le t \le 1$
   (b) $\mathbf{r}(t) = \langle -3t+3, 2t-1, -4t+2 \rangle, \quad 0 \le t \le 1$
   (c) $\mathbf{r}(t) = \langle 3\sin(t), 3\cos(t) \rangle, \quad \pi \le t \le 3\pi$
   (d) $\mathbf{r}(t) = \langle t^2, -t \rangle \quad -2 \le t \le 1$
2. $40\pi$
3. 15
4. $\sqrt{3}\pi$
5. 

| $(x,y)$ | $(-2,0)$ | $(-1,2)$ | $(0,-2)$ | $(1,1)$ | $(2,3)$ | $(3,2)$ | $(-1,0)$ | $(1,3)$ |
|---|---|---|---|---|---|---|---|---|
| $\mathbf{F}(x,y)$ | $\langle 0,-2\rangle$ | $\langle 2,-1\rangle$ | $\langle -2,0\rangle$ | $\langle 1,1\rangle$ | $\langle 3,2\rangle$ | $\langle 2,3\rangle$ | $\langle 0,-1\rangle$ | $\langle 3,1\rangle$ |

6. Without rescaling vectors to fit better:




With rescaling:




7. $\nabla f(x,y) = \langle x-y, y-x \rangle$

# Math 2551 Worksheet Answers: Vector Line Integrals

1. Top left: 0
   Bottom left: Negative
   Center: Positive
   Top right: 0
   Bottom right: Negative

2. 1

3. $\frac{25}{6}$

4. Circulation: $\frac{32}{3}$
   Flux: 0

5. Many possible examples: the top left and top right examples from 1), $\nabla f$ for any $f$ together with a closed curve $\mathbf{r}$, any example where $\mathbf{F} \cdot \mathbf{r}'(t) = 0$, i.e. the field and curve are orthogonal, and more.

# Math 2551 Worksheet Answers: Potentials and Conservative Vector Fields

1. $f(x, y) = \arctan(xy)$
2. $f(x, y, z) = x^2y - z^2y$ and $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = -16$.
3. $7 = a, 2 = c, 4 = d$, no restriction on $b$ or $e$.
4. $f(x, y, z) = xy^2z + x^2z^2$.
   $\int_C \mathbf{F} \cdot d\mathbf{r} = 5$
5. $2\pi$

# Math 2551 Worksheet Answers: Curl, Divergence, Green's Theorem

1. $\nabla \cdot \mathbf{F}$ is 0 in all quadrants.
   $(\nabla \times \mathbf{F}) \cdot \mathbf{k}$ is negative in all quadrants.

2. $\nabla \cdot \mathbf{F} = 0$
   $(\nabla \times \mathbf{F}) \cdot \mathbf{k} = -\frac{1}{4}$.

3. (a) $\mathbf{r}(t) = \langle 3\cos(t), 4\sin(t)\rangle$, $0 \le t \le 2\pi$.
   (b) Circulation: 0
   Flux: $48\pi$

4. Circulation: $44/15$
   Flux: The integrand is very difficult to work with, so we should not use Green's theorem here.

5. $-1/12$

6. One answer (linear algebra!) $\mathbf{r}(s,t) = s\langle 1,-1,0\rangle + t\langle 0,1,-1\rangle = \langle s, t-s, -t\rangle$, $s,t \in \mathbb{R}$.

# Math 2551 Worksheet Answers: Surfaces

1. One answer: $\mathbf{r}(u,v) = \langle u, 4-u^2, v\rangle$, $-2 \le u \le 2, 0 \le v \le 2$.
2. One answer: $\mathbf{r}(\phi,\theta) = \langle 2\sin(\phi)\cos(\theta), 2\sin(\phi)\sin(\theta), 2\cos(\phi)\rangle$ with $0 \le \phi \le \pi/6$ and $0 \le \theta \le 2\pi$.
   SA: $8\pi\left(1-\frac{\sqrt{3}}{2}\right)$
3. The tangent plane is $-\sqrt{2}(x-\sqrt{2}) - \sqrt{2}(y-\sqrt{2}) + 2(z-2) = 0$. This is the cone $z = \sqrt{x^2+y^2}$.
4. $\dfrac{2}{3}\pi(2^{3/2}-1)$
5. $\dfrac{\pi}{6}(5^{3/2}-1)$

# Math 2551 Worksheet Answers: Surface Integrals

1. 0
2. $\frac{10\pi}{3}$
3. 0

# Math 2551 Worksheet Answers: Stokes' Theorem

1. $H$ and $P$ have the same oriented boundary curve $C$ provided that they are oriented in the same way, so by Stokes' theorem the given integrals must be equal.
2. 0
3. 3

# Math 2551 Worksheet Answers: Review for Exam 3

1. $\int_0^{2\pi} \int_{2\pi/3}^{\pi} \int_2^5 \rho^4 \cos^2(\phi)\sin(\phi) d\rho\ d\phi\ d\theta$

2. (a) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{4-x^2} \ dz\ dy\ dx$

   (b) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{4-x^2} \ dz\ dy\ dx$

3. One solution: $\int_0^1 2(t)(-t^{3/2})^2 \sqrt{1+(\frac{3}{2}\sqrt{t})^2}\ dt.$

4. $f(x,y,z) = x^2 - \frac{1}{3}y^3 - 4\arctan(z).$

   Integral: $-\pi$.

5. $-4 \cdot \pi(2)^2$

6. $\pi a^2/4$

7. $\nabla \times \mathbf{F} = \langle 0-0, -(0-0), 0-0 \rangle$

8. $12\pi$

# Math 2551 Worksheet Answers: Review for Final

1. $3x+y+z=5$. There is not a unique plane because there is not a unique normal direction perpendicular to $\langle 3,1,1\rangle$.
2. $(0,5,24\pi)$
3. Domain $\{(x,y) \mid y \leq x^2\}$ Range $[0,\infty)$ Level curves are the parabolas $y = x^2 - c^2$ for all $c \geq 0$.
4. The limit does not exist
5. Tangent plane: $(x-1)+(y-1)-z=0$ Linearization: $L(x,y,z) = 1+(x-1)+(y-1)-z$
6. $\nabla f(1,2) = \langle 2,2\rangle$, $Df_{\mathbf{u}}(1,2) = 14/5$
7. $-\sin^2(1) - \sin(1)\cos(1) + \cos^2(1) + \cos(1)\cos(2) - 2\cos(1)\sin(2) - 2\sin(1)\sin(2)$
8. $(0,1)$ saddle point, $(2,1), (-2,1)$ local minimum
9. min: -32 at $(2,-2)$ max: 18 at $(1,1)$
10. min: -1/2 at $(1/\pm\sqrt{2}, 1/\pm\sqrt{2})$ and max: 1/2 at $(1/\pm\sqrt{2}, 1/\mp\sqrt{2})$.
11. $\displaystyle\int_{-3}^{3}\int_{0}^{\sqrt{9-4x^2}} y\,dy\,dx$
12. $\pi$
13. $(\bar{x},\bar{y}) = \left(\dfrac{1}{2-\ln(4)}, \dfrac{1}{2-\ln(4)}\right)$
14. $V = \dfrac{\pi}{6}(8\sqrt{2}-7)$
15. $\dfrac{64}{5}$
16. 0
17. 3
18. $1-e^{-2\pi}$
19. $\dfrac{3}{2}$
20. $\dfrac{208\pi}{5}$