

MATH 2551-D MIDTERM 1
VERSION A
FALL 2024
COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-2

Full name: _____

GT ID: _____

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

() I attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Show your work. Answers without work shown will receive little or no credit.
- Good luck! Write yourself a message of encouragement on the front page!

Question	Points
1	1
2	1
3	2
4	3
5	3
6	10
7	10
8	10
9	10
Total:	50

Learning Outcomes

- **G1: Lines.** Describe lines using the vector equation of a line. Find the intersections of lines. Describe the relationships of lines to each other. Solve problems with lines and planes.
- **G2: Planes.** Describe planes using the general equation of a plane. Find the intersections of planes. Find the equations of planes using a point and a normal vector. Describe the relationships of planes to each other. Solve problems with lines and planes.
- **G3: Tangent Vectors to Curves.** Compute tangent vectors to parametric curves; velocity, speed, and acceleration. Find equations of tangent lines to parametric curves.
- **G4: Arc Length.** Compute the arc length of a curve in two or three dimensions. Use arc length to solve problems.
- **G5: Curvature.** Compute normal vectors and curvature for curves in two and three dimensions. Interpret these objects geometrically and in application.
- **G6: Surfaces.** Graph cylinders from equations. Describe and find traces on surfaces. Identify standard quadric surfaces including: spheres, ellipsoids, elliptic paraboloid, hyperboloids, cones, hyperbolic paraboloid. Sketch quadric surfaces. Graph functions of two variables and determine their domains and ranges. Graph level curves.
- **G7: Limits of Functions.** Calculate the limits of functions of two variables or determine if they do not exist. Apply the Squeeze Theorem for functions of two variables. Define continuity of a function of two or more independent variables.

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Be sure to neatly fill in the bubble corresponding to your answer choice.

1. (1 point) [**G6: Surfaces**] The contour map for a hill and the contour map for a valley appear the same if the contours are not labeled.
 TRUE **FALSE**

2. (1 point) [**G3: Tangent Vectors to Curves**] A particle moving at constant speed must have zero acceleration.
 TRUE **FALSE**

3. (2 points) [**G3: Tangent Vectors to Curves**] If $\mathbf{r}(t)$ parameterizes a curve C in space, then _____ gives a direction vector for the tangent line to the curve at the point $\mathbf{r}(0)$.

4. (3 points) [**G2: Planes**] Which plane given below is **not** parallel to the plane $x + 2y - z = 3$?
 A) $x + 2y - z = 4$
 B) $3x + 6y - 3z = 3$
 C) The plane through the origin which is orthogonal to $\langle 1, 2, 1 \rangle$
 D) The plane through the origin which is orthogonal to $\langle -1, -2, 1 \rangle$
 E) The plane containing the points $(3, 0, 0)$, $(0, 3/2, 0)$, and $(0, 0, -3)$.

5. (3 points) [**G5: Curvature**] Which curve has the greatest curvature at the indicated point?
 A) The line $\ell(t) = \langle t + 1, 3t + 3, 5t - 5 \rangle$ at the point $(2, 6, 0)$.
 B) The circle $x^2 + y^2 = 1$ at the point $(1, 0)$
 C) The circle $x^2 + y^2 = 4$ at the point $(\sqrt{2}, \sqrt{2})$
 D) A curve with constant curvature $\kappa = \frac{1}{2}$ at any point on the curve.
 E) No single curve above has the greatest curvature at the indicated point.

6. [G1: Lines], [G2: Planes] Consider the plane p with equation $-2x + y + 4z = 2$.

(a) (2 points) Give a line ℓ through the point $(1, 1, 1)$ which is orthogonal to p .

(b) (4 points) Give a different line through the point $(1, 1, 1)$ which is orthogonal to ℓ .
Justify why your line is orthogonal to ℓ .

(c) (4 points) Give a third line which is skew to ℓ (it is not parallel to ℓ and does not intersect ℓ). Justify why your line is skew to ℓ .

7. [**G4: Arc Length**] Consider the curve C which is the part of a helix between $(2, 0, 0)$ and $(2, 0, 4\pi)$ parameterized by

$$\mathbf{r}(t) = \langle 2 \cos(t), 2 \sin(t), t \rangle, \quad 0 \leq t \leq 4\pi.$$

- (a) (7 points) Compute the arc length function for C based at $(0, 2, \pi/2)$.

- (b) (3 points) Use your arc length function to compute the length of the section of the curve between $(-2, 0, \pi)$ and $(0, 2, 5\pi/2)$. Fully simplify your answer.

8. [G6: Surfaces]

(a) (6 points) In this part, you will consider the surface with equation $y - x^2 - z^2 = 1$. Fill in each blank with the appropriate letter.

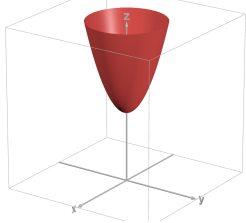
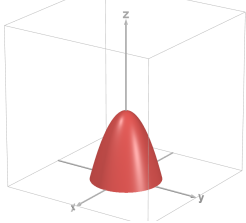
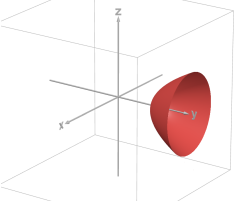
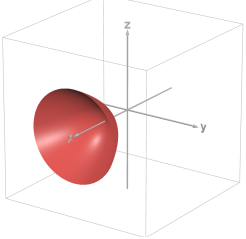
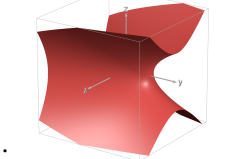
i. The cross sections in the planes $x = k$ are _____

ii. The cross sections in the planes $y = k$ are _____

iii. The cross sections in the planes $z = k$ are _____

iv. The quadric surface is a _____

v. The surface looks most like graph _____

Cross Section Choices	Quadric Surface Choices	Graph Choices
A: Ellipses/Circles	E: Ellipsoid	K: 
B: Lines	F: Elliptical Paraboloid	L: 
C: Parabolas	G: Hyperbolic Paraboloid	M: 
D: Hyperbolas	H: Hyperboloid of 1 or 2 Sheets	N: 
	I: Cone	O: 

- (b) (4 points) Find and sketch the domain of the function $f(x, y) = \frac{\sqrt{4 - (x - 2)^2 - y^2}}{x^2 e^y}$.
Be sure to clearly indicate which points on the boundary are included or excluded.

9. [G7: Limits of Functions]

- (a) (4 points) Compute

$$\lim_{(x,y) \rightarrow (1,2)} \frac{2x - y}{2x^2 + xy - y^2}$$

or show that this limit does not exist.

(b) (2 points) Find a value C so that the function $h(x, y) = \frac{3x^2}{x^2 + y^2}$ satisfies $|h(x, y)| \leq C$.

(c) (4 points) Use the Squeeze Theorem to show that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y^3}{x^2y^2 + y^4} = 0.$$

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FORMULA SHEET

- Dot product: $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1v_1 + u_2v_2 + u_3v_3$
- Dot product magnitudes: $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos(\theta)$
- Cross product: $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- Cross product magnitude: $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin(\theta)$
- Arc length: $L = \int_a^b \|\mathbf{r}'(t)\| dt$
- Arc length function: $s(t) = \int_{t_0}^t \|\mathbf{r}'(T)\| dT$
- Unit tangent vector: $\mathbf{T} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{d\mathbf{r}}{ds}$
- Curvature: $\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\| = \frac{1}{\|\mathbf{v}\|} \left\| \frac{d\mathbf{T}}{dt} \right\| = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3}$
- Principal unit normal vector: $\mathbf{N} = \frac{1}{\kappa} \frac{d\mathbf{T}}{ds} = \frac{d\mathbf{T}/dt}{\|d\mathbf{T}/dt\|}$
- Two-Path Test: If there exist two different paths C_1 and C_2 through the point (a, b) along which the limit of $f(x, y)$ takes on different values, then

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y)$$

does not exist.

- Squeeze theorem: If $f(x, y) = g(x, y)h(x, y)$, where

$$\lim_{(x,y) \rightarrow (a,b)} g(x, y) = 0 \quad \text{and} \quad |h(x, y)| \leq C \text{ near } (a, b)$$

for some constant $C > 0$, then

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y) = 0.$$