MATH 2551-K MIDTERM 1
VERSION A
SPRING 2024
COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-14.2

Full name: ______________________________ **GT ID:**____________________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work.
- Good luck! Write yourself a message of encouragement on the front page!

Question	Points
1	2
2	2
3	2
4	4
5	10
6	10
7	10
8	10
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Be sure to neatly fill in the bubble corresponding to your answer choice.

1. (2 points) If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, then $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$.

   ○ **TRUE** ○ **FALSE**

2. (2 points) The domain of the function $f(x, y) = \sqrt{y-3} + xe^x$ is $[3, \infty)$.

   ○ **TRUE** ○ **FALSE**

3. (2 points) A hummingbird with velocity vector $\mathbf{r}'(t)$ starts flying from $(2, 1, -3)$ at time $t = 0$ and flies around for 5 seconds. Where is the hummingbird located at time $t = 5$ if $\int_0^5 \mathbf{r}'(t)\, dt = 0$? You do not need to show your work on this problem.

4. (4 points) Find the value of the limit below or show that it does not exist. To receive full credit, your answer must show work supporting your answer and if you show it does not exist, mention the test that you are using.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$$

5. In this problem, you will work with the vector-valued function $\mathbf{r}(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$, defined for $t \in \mathbb{R}$.

   (a) (2 points) Find the two values of t where $\mathbf{r}(t)$ is $\langle 0, 1, 1 \rangle$ and $2\sqrt{2}\mathbf{i} + e^2\mathbf{j} + \frac{1}{e^2}\mathbf{k}$, respectively.

   (b) (8 points) Find the length of the curve $\mathbf{r}(t)$ between $(0, 1, 1)$ and $(2\sqrt{2}, e^2, \frac{1}{e^2})$. Show all of your work and fully simplify your final answer.

6. In this problem, you will work with the spiral plane curve

$$\mathbf{r}(t) = \langle \cos(t) + t\sin(t), \sin(t) - t\cos(t) \rangle$$

for $t > 0$.

(a) (5 points) Compute the unit tangent vector $\mathbf{T}(t)$.

(b) (3 points) Compute the principal unit normal vector $\mathbf{N}(t)$.

(c) (2 points) Compute the curvature $\kappa(t)$.

7. Let L_1 be the line parameterized by $\mathbf{r}_1(t) = \langle 1, 2, 3\rangle t + \langle 1, -4, -4\rangle$ and L_2 be the line parameterized by $\mathbf{r}_2(p) = \langle 1, 1, 1\rangle + p\langle 2, -1, 1\rangle$.

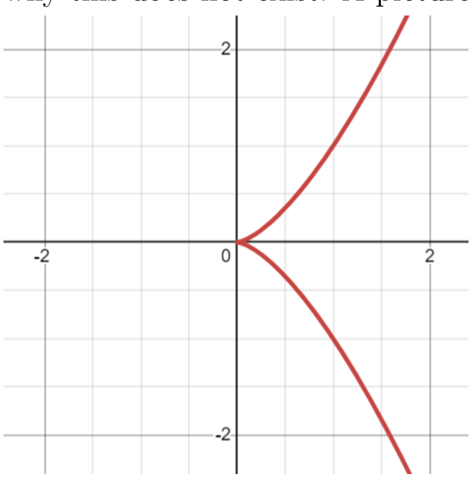
   (a) (4 points) Show that these lines are not skew (i.e. they are either parallel or intersect).

   (b) (5 points) Find an equation for the unique plane which contains both lines. Give your answer in the form $Ax + By + Cz = D$.

   (c) (1 point) Why is part (a) necessary for the problem? In other words, why would part (b) be impossible if the lines were skew?

8. (a) (6 points) Let $f(x, y) = \dfrac{\ln(x - y)}{\sqrt{4 - x^2 - y^2}}$. Graph the domain of f on the provided axes below. Be sure to show all your work in finding the domain. Indicate whether or not each part of the boundary of the domain is included.

(b) (4 points) Compute the tangent line for the curve $\mathbf{r}(t) = \langle t^2, t^3 \rangle$ at $t = 0$ or explain why this does not exist. A picture of the curve is below.



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FORMULA SHEET

- $\langle u_1, u_2, u_3\rangle \cdot \langle v_1, v_2, v_3\rangle = u_1v_1 + u_2v_2 + u_3v_3$
- $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$
- $\langle u_1, u_2, u_3\rangle \times \langle v_1, v_2, v_3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \int_a^b |\mathbf{r}'(t)|\ dt$
- $s(t) = \int_{t_0}^t |\mathbf{r}'(T)|\ dT$
- $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{d\mathbf{r}}{ds}$
- $\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \frac{1}{\kappa}\frac{d\mathbf{T}}{ds} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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