**MATH 2551-K MIDTERM 1**
**VERSION A**
**FALL 2023**
**COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1**

**Full name:** _____ **GT ID:** _____

**Honor code statement:** I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(  ) I attest to my integrity.

**Read all instructions carefully** before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work.
- Good luck! Write yourself a message of encouragement on the front page!

| Question | Points |
|---|---|
| 1 | 2 |
| 2 | 2 |
| 3 | 2 |
| 4 | 4 |
| 5 | 10 |
| 6 | 10 |
| 7 | 10 |
| 8 | 10 |
| Total: | 50 |

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Be sure to neatly fill in the bubble corresponding to your answer choice.

1. (2 points) Any plane has only two distinct normal vectors.

   ◯ **TRUE** ◯ **FALSE**

2. (2 points) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$, then $\mathbf{u} \times \mathbf{v} + \mathbf{v} \times \mathbf{u} = \mathbf{0}$.

   ◯ **TRUE** ◯ **FALSE**

3. (2 points) Order the following curves in increasing order of curvature.

   A) A helix of curvature 2

   B) A line segment of length 10

   C) A circle of radius 20

4. (4 points) Which of the following vectors could be the principal unit normal vector at time $t = 2$ to a curve whose tangent line at $t = 2$ is given by

$$\ell(p) = \langle 1, 0, -1 \rangle + p\langle 1, 2, 1 \rangle.$$

   You must justify your answer to receive full credit.

   A) $\langle 1, 1, 1 \rangle$

   B) $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

   C) $\langle -2, 1, 0 \rangle$

   D) $\left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right\rangle$

5. (10 points) Tobert, a hard-working documentarian, is hiking through the African savannah for two days and his path is given by the curve $\mathbf{r}(t) = \langle t^2, \frac{1}{3}t^3, 2\rangle$ for $0 \leq t < 48$, where $t$ is measured in hours and $\mathbf{r}(t)$ in meters.

   (a) What is Tobert's position two hours into his hike?

   (b) Find a function $s(t)$ that gives the total distance Tobert has hiked for any time $t$ since he started at $t = 0$.

   (c) How far has Tobert hiked when he is at the point $(4, 8/3, 2)$?

6. In this problem, you will work with the differential equation

$$\mathbf{r}''(t) = 3\mathbf{i} + e^t\mathbf{j} + 49e^{7t}\mathbf{k}, \quad -\infty < t < \infty.$$

(a) (6 points) Find all vector-valued functions $\mathbf{r}(t)$ with $-\infty < t < \infty$ which are solutions to this equation.

*Hint: Your answer to this part should include some undetermined constant vectors.*

(b) (2 points) Find all vector-valued functions $\mathbf{r}(t)$ with $-\infty < t < \infty$ which are solutions to this equation and also have the property that $\mathbf{r}'(0) = 3\mathbf{j} + 7\mathbf{k}$.

(c) (2 points) Find all vector-valued functions $\mathbf{r}(t)$ with $-\infty < t < \infty$ which are solutions to this equation, have the property that $\mathbf{r}'(0) = 3\mathbf{j} + 7\mathbf{k}$ and also have the property that $\mathbf{r}(0) = \langle 1, 1, 5\rangle$.

7. Let $p_1$ be the plane defined by the equation $2x - 3y + z = -10$ and let $Q$ be the point $(3, 1, 1)$.

   (a) (2 points) Find an equation for the plane $p_2$ which contains the point $Q$ and is parallel to $p_1$.

   (b) (2 points) Find an equation for the line $\ell$ which passes through $Q$ and is orthogonal to both planes.

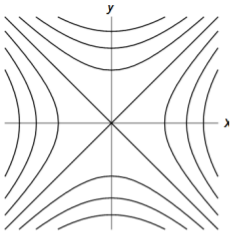
   (c) (3 points) Find the point $R$ where the line $\ell$ intersects the plane $p_1$.

   (d) (3 points) Compute the distance between the two planes using your work in parts (a)-(c) above.

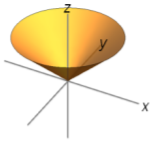
8. (a) (4 points) Find and sketch the domain of the function $f(x,y) = \sqrt{4 - x^2 - y^2}$. Show your work and label your axes clearly.

(b) (6 points) Match the contour plots and graphs below with the given functions of two variables.

| Function | Graph | Contour Plot |
|---|---|---|
| $f(x,y) = \cos\left(\sqrt{x^2+y^2}\right)$ | _____ | _____ |
| $g(x,y) = x^2 - y^2$ | _____ | _____ |
| $h(x,y) = x^2 + 2y^2$ | _____ | _____ |




A

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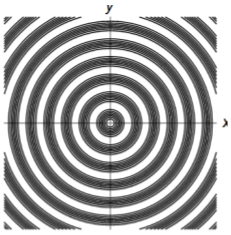


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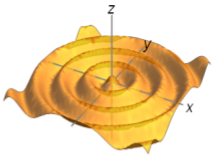


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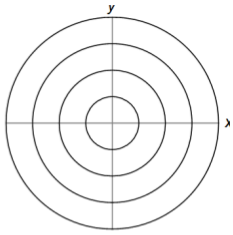


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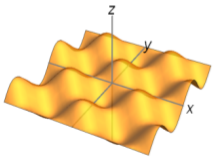


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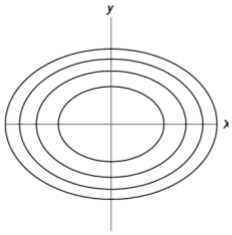


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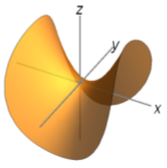


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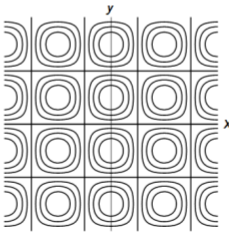
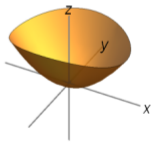



I




J

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## FORMULA SHEET

- $\langle u_1, u_2, u_3\rangle \cdot \langle v_1, v_2, v_3\rangle = u_1v_1 + u_2v_2 + u_3v_3$
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$
- $\langle u_1, u_2, u_3\rangle \times \langle v_1, v_2, v_3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \displaystyle\int_a^b |\mathbf{r}'(t)|\, dt$
- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(T)|\, dT$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right| = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa}\dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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