# MATH 2551-D MIDTERM 3
# VERSION A
# SPRING 2023
# COVERS SECTIONS 15.1-15.8

**Full name:** ______________________________ **GT ID:**____________________

**Honor code statement:** I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(    ) I attest to my integrity.

**Read all instructions carefully** before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to take the exam.
- You may not use aids of any kind.
- Answers with little or no work shown will receive little or no credit.
- Good luck! Write yourself a message of encouragement on the front page!

| Question | Points |
|---|---|
| 1 | 2 |
| 2 | 2 |
| 3 | 2 |
| 4 | 3 |
| 5 | 8 |
| 6 | 9 |
| 7 | 14 |
| 8 | 10 |
| Total: | 50 |

For problems 1-3 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Be sure to neatly fill in the bubble corresponding to your answer choice.

1. (2 points)

$$\int_5^6 \int_7^8 x^3 e^y \, dy \, dx = \left( \int_5^6 x^3 \, dx \right) \left( \int_7^8 e^y \, dy \right)$$

○ **TRUE** ○ **FALSE**

2. (2 points) If $g(x, y)$ is strictly positive at all points in $\mathbb{R}^2$, then

$$\iint_{R_1} g(x, y) \, dA > \iint_{R_2} g(x, y) \, dA$$

if the area of $R_1$ is larger than the area of $R_2$.

○ **TRUE** ○ **FALSE**

3. (2 points) The integral below might compute the total mass of a solid sphere of radius 1 centered at the origin which has constant density $\delta = 2$.

$$\int_0^{2\pi} \int_0^1 2r \, dr \, d\theta$$

○ **TRUE** ○ **FALSE**

4. (3 points) Suppose the contour plot shown gives the height of a pile of dirt in feet. Which of the following is the **best** lower bound for the volume of dirt in the hill?

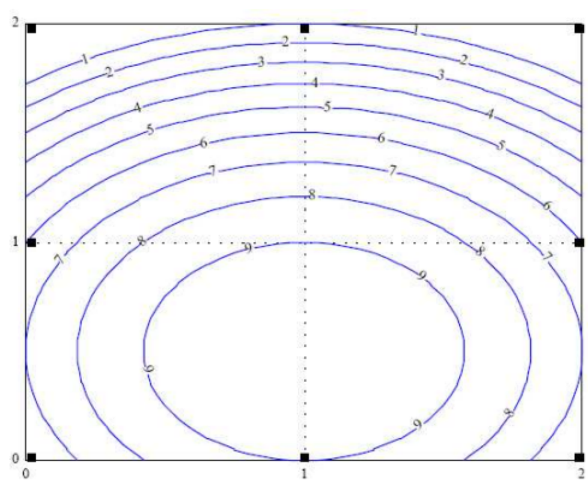
○ **A)** 0*1+0*1+6*1+6*1

○ **B)** 9*1+9*1+9*1+9*1

○ **C)** 9

○ **D)** 6*1+6*1+9*1+9*1

○ **E)** 0



5. The temperature in degrees Celsius of a thin rectangular metal plate located on the region $[0, 1] \times [1, 4]$ is given by

$$T(x, y) = (x^2 + \sqrt{y}) \sin(x^2 y^2).$$

Finding the average temperature on the plate exactly is very difficult, but we can find an upper bound for it.

(a) (2 points) Find the area of the plate.

(b) (2 points) Write an expression including a double integral for the average temperature on the plate.

(c) (4 points) Show that the average temperature on the plate is at most 3 degrees Celsius. *Hint: It may be useful to consider the maximum temperature on the plate.*

6. (a) (4 points) Sketch the region of integration for and give an interpretation of the value of the integral expression below.

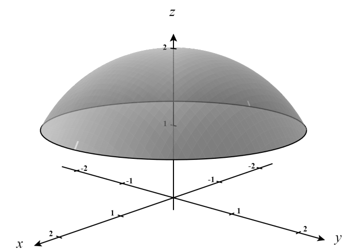
$$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} dx\ dy + \int_1^4 \int_{y-2}^{\sqrt{y}} dx\ dy$$

(b) (3 points) Write a new integral expression with one or more integrals for the same value, written in the opposite order of integration.

(c) (2 points) Explain which expression you would rather evaluate. You do **not** need to actually evaluate either integral.

*Your interpretation and your explanation should each be a full sentence.*

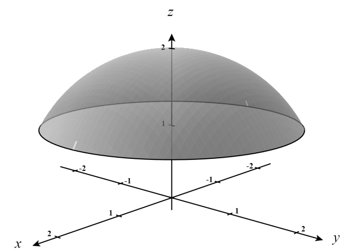
7. Let $D$ be the smaller cap cut from a solid ball of radius 2 units by the plane $z = 1$.




(a) (4 points) Express the volume of $D$ as an iterated triple integral in spherical coordinates. Show all of your work in setting up this integral. *You do not need to evalute the integral.*

(b) (4 points) Express the volume of $D$ as an iterated triple integral in cylindrical coordinates. Show all of your work in setting up this integral. *You do not need to evalute the integral.*

Let $D$ be the smaller cap cut from a solid ball of radius 2 units by the plane $z = 1$.



(c) (4 points) Express the volume of $D$ as an iterated triple integral in Cartesian coordinates. Show all of your work in setting up this integral. *You do not need to evalute the integral.*

(d) (2 points) Use one of parts (a), (b), or (c) to find the volume of $D$. Fully simplify your answer.

8. (10 points) Make an appropriate change of variables and evaluate the integral

$$\iint_R (x+y)e^{x^2-y^2}\, dA,$$

where $R$ is the rectangle enclosed by the lines $x-y=0, x-y=2, x+y=0$, and $x+y=3$. Include a sketch of the new region of integration $G$ in the $uv$-plane.

Hint: Think about what you would like $u(x,y)$ and $v(x,y)$ to be and work backwards to find the transformation $\mathbf{T}(u,v)$.

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## FORMULA SHEET

- Area/volume: area$(R) = \iint_R dA$, volume$(D) = \iiint_D dV$
- Trig identities: $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$, $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$
- $f_{avg} = \dfrac{\iint_R f(x,y)dA}{\text{area of } R}$, or $f_{avg} = \dfrac{\iiint_D f(x,y,z)dV}{\text{volume of } D}$
- Mass: $M = \iint_D \delta\ dA$ or $M = \iiint_D \delta\ dV$
- First moments (2D plate): $M_y = \iint_R x\delta\ dA$, $M_x = \iint_R y\delta\ dA$
- Center of mass (2D plate): $(\bar{x}, \bar{y}) = \left(\dfrac{M_y}{M}, \dfrac{M_x}{M}\right)$
- First moments (3D solid): $M_{yz} = \iiint_D x\delta\ dV$, $M_{xz} = \iiint_D y\delta\ dV$, $M_{xy} = \iiint_D z\delta\ dV$
- Center of mass (3D solid): $(\bar{x}, \bar{y}, \bar{z}) = \left(\dfrac{M_{yz}}{M}, \dfrac{M_{xz}}{M}, \dfrac{M_{xy}}{M}\right)$
- Polar coordinates: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $dA = r\ dr\ d\theta$
- Cylindrical coordinates: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = z$, $dV = r\ dz\ dr\ d\theta$
- Spherical coordinates: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$, $dV = \rho^2\sin(\phi)\ d\rho\ d\phi\ d\theta$
- Substitution for double integrals: If $R$ is the image of $G$ under a coordinate transformation $\mathbf{T}(u,v) = \langle x(u,v), y(u,v)\rangle$ then

$$\iint_R f(x,y)\ dx\ dy = \iint_G f(\mathbf{T}(u,v))|\det D\mathbf{T}(u,v)|\ du\ dv.$$

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