MATH 2551-K/L EXAM 3
VERSION A
FALL 2022
COVERS SECTIONS 15.1-15.8

Full name: ______________________________ **GT ID:**____________________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam.

(   ) I attest to my integrity.

Read all instructions carefully before beginning.

- You have 50 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work.
- Good luck! Write yourself a message of encouragement on the front page!

Question	Points
1	10
2	9
3	7
4	5
5	10
6	9
Total:	50

FORMULA SHEET

- $\sin^2(x) = \frac{1}{2}(1-\cos(2x)), \quad \cos^2(x) = \frac{1}{2}(1+\cos(2x))$
- $f_{avg} = \dfrac{\iint_R f(x,y)dA}{\iint_R dA}, \qquad f_{avg} = \dfrac{\iiint_D f(x,y,z)dV}{\iiint_D dV}$
- $x = r\cos(\theta), \quad y = r\sin(\theta)$
- $r^2 = x^2 + y^2, \quad \tan(\theta) = \dfrac{y}{x}$
- $dA = dx\ dy = r\ dr\ d\theta$
- $M = \iiint_D \rho(x,y,z)\ dV$
- $M_{yz} = \iiint_D x\rho(x,y,z)\ dV, \quad M_{xz} = \iiint_D y\rho(x,y,z)\ dV, \quad M_{xy} = \iiint_D z\rho(x,y,z)\ dV$
- $I_x = \iiint_D (y^2+z^2)\rho(x,y,z)\ dV$
- $I_y = \iiint_D (x^2+z^2)\rho(x,y,z)\ dV$
- $I_z = \iiint_D (x^2+y^2)\rho(x,y,z)\ dV$
- $(\bar{x},\bar{y},\bar{z}) = \left(\dfrac{M_{yz}}{M}, \dfrac{M_{xz}}{M}, \dfrac{M_{xy}}{M}\right)$
- $x = r\cos(\theta), \quad y = r\sin(\theta), \quad z = z$
- $r^2 = x^2 + y^2, \quad \tan(\theta) = \dfrac{y}{x}, \quad z = z$
- $x = \rho\sin(\phi)\cos(\theta), \quad y = \rho\sin(\phi)\sin(\theta), \quad z = \rho\cos(\phi)$
- $\rho^2 = x^2 + y^2 + z^2, \quad \tan(\phi) = \dfrac{\sqrt{x^2+y^2}}{z}, \quad \tan(\theta) = \dfrac{y}{x}$
- $r = \rho\sin(\phi), \quad \theta = \theta, \quad z = \rho\cos(\phi)$
- $\rho^2 = r^2 + z^2, \quad \tan(\phi) = \dfrac{r}{z}, \quad \theta = \theta$
- $dV = dx\ dy\ dz = r\ dz\ dr\ d\theta = \rho^2\sin(\phi)\ d\rho\ d\phi\ d\theta$
- If $\mathbf{T} : G \to R$ is a 1-to-1 differentiable function with $\mathbf{T}(u,v) = \langle x(u,v), y(u,v)\rangle$ and $f(x,y)$ is continuous on R then

$$\iint_R f(x,y)\ dx\ dy = \iint_G f(\mathbf{T}(u,v))|D\mathbf{T}(u,v)|\ du\ dv.$$

1. Choose whether the following statements are true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false.

   (a) (2 points) The cylindrical coordinates of the point with Cartesian coordinates $(4, \pi, 0)$ are $(-4, 0, 0)$.

   ◯ **TRUE** ◯ **FALSE**

   (b) (2 points) The equation $r = 2\sin(\theta)$ describes a circle.

   ◯ **TRUE** ◯ **FALSE**

   (c) (2 points) The mass of a region in the plane might be given by $\int_0^x \int_0^y 3\ dy\ dx$.

   ◯ **TRUE** ◯ **FALSE**

   (d) (2 points)

   $$\int_0^1 \int_0^1 1\ dy\ dx > \int_0^1 \int_x^1 1\ dy\ dx.$$

   ◯ **TRUE** ◯ **FALSE**

   (e) (2 points) The region $0 \leq \rho \leq 4, 0 \leq \phi \leq \pi, 0 \leq \theta \leq \pi$ describes the bottom half of a sphere of radius 4 centered on the origin.

   ◯ **TRUE** ◯ **FALSE**

2. (9 points) Sketch the domain of integration corresponding to

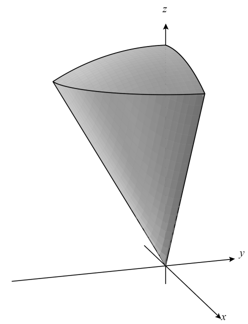
$$\int_0^1 \int_{x^{2/3}}^1 xe^{y^4} dy\ dx.$$

Then change the order of integration and evaluate. Explain the simplification achieved by changing the order.

3. (7 points) Compute the average distance to the origin among points in a disk D of radius 3 centered at the origin in $\mathbb{R}^2$, $D = \{(x, y) \mid x^2 + y^2 \leq 9\}$.

4. (5 points) Suppose that the mass of a thin metal plate D in the shape of a disk of radius 1 m centered at the origin is 12 kg and the center of mass of this disk is the point $(0, 1/2)$. Is it possible that the density $\rho(x, y)$ of the disk depends only on x and not on y, i.e. $\rho = f(x)$? Give an explanation for your answer. It may help to consider $\bar{y}$.

5. Let D be the solid in $\mathbb{R}^3$ which is bounded below by the cone $z = \sqrt{3x^2 + 3y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 16$ and lies in the fourth octant ($x \geq 0, y \leq 0, z \geq 0$). The projection of D to the xy-plane is the portion of the disk $x^2 + y^2 \leq 4$ in the fourth quadrant.



   (a) (5 points) Write an integral in cylindrical coordinates for the volume of D. Do not evaluate your integral.

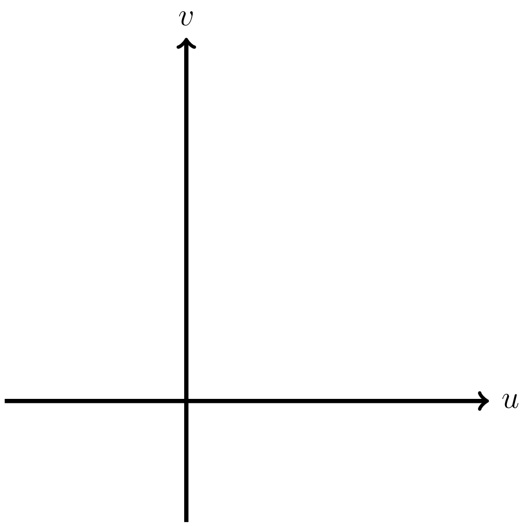
   (b) (5 points) Write an integral in spherical coordinates for the volume of D. Do not evaluate your integral.

6. In this problem you will compute the integral

$$\iint_R (2x+y)(x-y)\ dx\ dy$$

for the region R in the first quadrant bounded by the lines $2x+y=4, 2x+y=7, x-y=2,$ and $x-y=-1$ using the transformation $u=x-y, v=2x+y$.

(a) (3 points) Transform the given bounds into equations in u and v. Use these to sketch the region G in the uv-plane that is the image of R under this transformation.



(b) (4 points) Solve the transformation equations for x and y in terms of u and v and compute the Jacobian determinant, i.e. find $\mathbf{T}(u,v)$ and $|D\mathbf{T}(u,v)|$. If you need more space, use the top of the next page.

(c) (2 points) (Problem 6 continued) Use your results from (a) and (b) to use change of variables with $u = x - y, v = 2x + y$ to rewrite the integral

$$\iint_R (2x + y)(x - y)\ dx\ dy$$

as an integral with respect to u and v over the region G. Do not evaluate your integral.