MATH 2551-K/L EXAM 2
VERSION A
FALL 2022
COVERS SECTIONS 14.1-14.8

Full name: ______________________________ **GT ID:**____________________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam.

(   ) I attest to my integrity.

Read all instructions carefully before beginning.

- You have 50 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work.
- Good luck! Write yourself a message of encouragement on the front page!

Question	Points
1	10
2	10
3	10
4	10
5	10
Total:	50

FORMULA SHEET

- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$
- $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- For $\mathbf{f}(x_1, \ldots, x_n) = \langle f_1(x_1, \ldots, x_n), \ldots, f_m(x_1, \ldots, x_n)\rangle$

$$D\mathbf{f} = \begin{bmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \cdots & (f_1)_{x_n} \\ (f_2)_{x_1} & (f_2)_{x_2} & \cdots & (f_2)_{x_n} \\ \vdots & \ddots & \cdots & \vdots \\ (f_m)_{x_1} & (f_m)_{x_2} & \cdots & (f_m)_{x_n} \end{bmatrix}$$

- At $\mathbf{a}$, $L(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} - \mathbf{a})$
- If $h = g(f(\mathbf{x}))$ then $Dh(\mathbf{x}) = Dg(f(\mathbf{x}))Df(\mathbf{x})$
- If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f(P) = Df(P)\mathbf{u} = \nabla f(P) \cdot \mathbf{u}$
- For $f(x, y)$, $Hf(x, y) = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$
- If (a, b) is a critical point of $f(x, y)$ then
  1. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) < 0$ then f has a local maximum at (a, b)
  2. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) > 0$ then f has a local minimum at (a, b)
  3. If $\det(Hf(a, b)) < 0$ then f has a saddle point at (a, b)
  4. If $\det(Hf(a, b)) = 0$ the test is inconclusive
- If (a, b) is the location of a min or max of $f(x, y)$ subject to the constraint $g(x, y) = c$, then $\nabla f(a, b) = \lambda \nabla g(a, b)$ for some λ.

1. Choose whether the following statements are true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false.

   (a) (2 points) Suppose we have a function $f(x, y)$ and $\mathbf{u} \in \mathbb{R}^2$ is a unit vector. Then $D_{\mathbf{u}}f(a, b)$ is a vector.

   ○ **TRUE** ○ **FALSE**

   (b) (2 points) If $f(x, y) = (x + y)^{50}$, then all of the 100^{th} order partial derivatives of f will be 0.

   ○ **TRUE** ○ **FALSE**

   (c) (2 points) The total derivative Df of the function $f(x, y, z, w) = 2xyz + 3\ln(w) + \sin(e^{xw+z})$ is represented by a 1×4 matrix.

   ○ **TRUE** ○ **FALSE**

   (d) (2 points) Suppose $\lim_{(x,y)\to(0,0)} f(x, y) = 1$ along the y-axis, the x-axis, and the line $y = x$ and $f(0, 0) = 1$. Then f must be continuous at $(0, 0)$.

   ○ **TRUE** ○ **FALSE**

   (e) (2 points) The domain of the function $f(x, y) = \sqrt{x - 2} + y$ is the interval $[2, \infty)$.

   ○ **TRUE** ○ **FALSE**

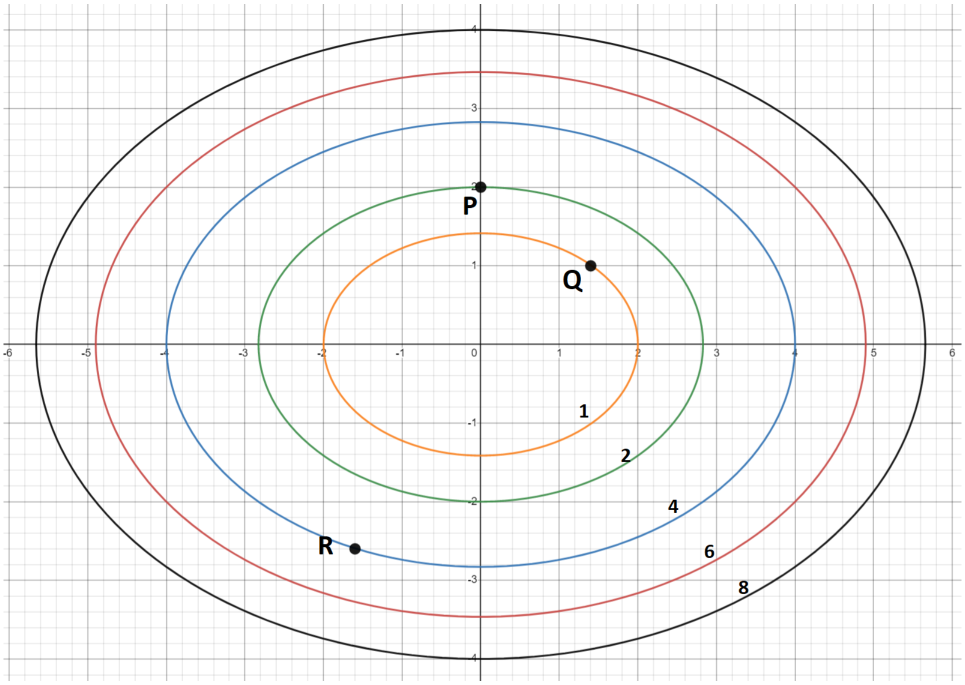
2. Let $f(x, y, z) = x^2 + z^2 e^{y-x}$.

   (a) (5 points) Find an equation of the tangent plane to the level surface $f = 13$ at the point $P = (2, 3, \frac{3}{\sqrt{e}})$.

   (b) (3 points) Find the linearization $L(x, y, z)$ of f at P.

   (c) (2 points) Use the linearization you found to approximate the value of $f(2, 3, \frac{4}{\sqrt{e}})$

3. Below is a contour plot for a smooth function $f(x, y)$. Use this contour plot to answer parts (a)-(c) below.



(a) (2 points) Determine the sign $(+, -, 0)$ of f_x at the point Q.

Answer:

(b) (2 points) Determine the sign $(+, -, 0)$ of the directional derivative of f in the direction $\mathbf{i} + \mathbf{j}$ at the point R.

Answer:

(c) (2 points) Draw a vector in the direction of greatest increase of f at the point P.

(d) (4 points) If $g(x, y) = 2x^2 - y^2$ and $\langle x(t), y(t) \rangle = \langle \cos(t), \sin(t) \rangle$, compute $\frac{dg}{dt}$ at $t = \pi/3$.

Answer:

4. For all parts of this question be sure to justify your conclusion, using at least one complete sentence.

   (a) (4 points) Suppose $f(x, y)$ is a differentiable function defined on all of $\mathbb{R}^2$, $Df(1,3) = \begin{bmatrix} 0 & 0 \end{bmatrix}$, $f_{xx}(1,3) = -2$, $f_{xy}(1,3) = f_{yx}(1,3) = 4$, and $f_{yy}(1,3) = -10$.

   What can you conclude about the behavior of f at $(1,3)$?

   (b) (6 points) Let $g(x, y) = 2x^2 + y^2$ defined on the disk $R = \{(x, y) \mid x^2 + y^2 \leq 1\}$. Determine the absolute extreme values of g on R.

5. (10 points) You are traveling on an airplane and would like to know what the largest carry-on suitcase you can bring with you is. The airline requires that any carry-on item has linear size (sum of the length, width, and height) at most 72 inches. Find the dimensions and maximum volume of the box-shaped suitcase with linear size 72 inches assuming that all of the dimensions must be non-negative. Be sure to explain how you know you have found a maximum.