Math 2551-G Quiz 3

QUIZ KEY

You have 10 minutes to take the Quiz. You may not use aids of any kind.

Learning Targets

D1: Computing Derivatives. I can compute partial derivatives, total derivatives, directional derivatives, and gradients. I can use the Chain Rule for multivariable functions to compute derivatives of composite functions.

Tasks

1. Answer the True/False questions below by writing **T** or **F** in the box. Answer the Fill-In question by writing the missing item into the box. You must get three of them correct to receive a **Success**.

   (a) **True/False:** Let $f : \mathbb{R}^7 \to \mathbb{R}^9$ be a differentiable function. Then the total derivative of f at the point $\mathbf{p}$ is a 9×7 matrix.

   T

   (b) **True/False:** Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function. Then the rate of change of f at the point (a, b) in the direction of the gradient of f at (a, b) is zero.

   F

   (c) **True/False:** For every function $f : \mathbb{R}^3 \to \mathbb{R}$, the rate of change of f at (a, b, c) in the direction of a vector $\mathbf{v}$ is given by $Df(a, b, c) \cdot \mathbf{v}$.

   F

   (d) **Fill-In:** Chain Rule: If $g : \mathbb{R}^n \to \mathbb{R}^p$ and $f : \mathbb{R}^p \to \mathbb{R}^m$ are differentiable functions and $h : \mathbb{R}^n \to \mathbb{R}^m$ is the composite function $h(\mathbf{x}) = f(g(\mathbf{x}))$, then

   $$Dh(\mathbf{x}) = \underline{\hspace{6cm}}.$$

   $Df(g(\mathbf{x})) \cdot Dg(\mathbf{x})$

Quiz continues on the back

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2. Compute the total derivative of the composite function $z(t) = f(\mathbf{r}(t))$, where

$$\mathbf{r}(t) = \begin{bmatrix} e^t \\ t^3 + 2t \\ \sin(t) \end{bmatrix} \qquad \text{and} \qquad f(x, y, z) = 2x + xe^{yz}$$

at the point $t = 0$.

Solution. We have $D\mathbf{r}(t) = \begin{bmatrix} e^t \\ 3t^2 + 2 \\ \cos(t) \end{bmatrix}$ and so $D\mathbf{r}(0) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$. Next, we compute $Df(x, y, z)$:

$$Df(x, y, z) = \begin{bmatrix} 2 + e^{yz} & xze^{yz} & xye^{yz} \end{bmatrix}.$$

Thus, $Df(\mathbf{r}(0)) = Df(1, 0, 0) = \begin{bmatrix} 3 & 0 & 0 \end{bmatrix}$. Finally, we have

$$Dz(0) = Df(\mathbf{r}(0))D\mathbf{r}(0) = \begin{bmatrix} 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 3.$$