Math 2551-G Quiz 1

QUIZ KEY

You have 10 minutes to take the Quiz. You may not use aids of any kind.

Learning Targets

G1: Lines and Planes. I can describe lines using the vector equation of a line. I can describe planes using the general equation of a plane. I can find the equations of planes using a point and a normal vector. I can find the intersections of lines and planes. I can describe the relationships of lines and planes to each other. I can solve problems with lines and planes.

Tasks

1. Answer the True/False questions below by writing **T** or **F** in the box. You must get three of them correct to receive a **Success**.

   (a) **True/False:** There is exactly one possible parameterization for any line in space.

   $\boxed{\text{False}}$

   (b) **True/False:** Any three non-collinear points in 3-space determine a unique plane.

   $\boxed{\text{True}}$

   (c) **True/False:** If two lines are both orthogonal to a third line, then they are parallel to each other.

   $\boxed{\text{False}}$

   (d) **True/False:** Two lines in $\mathbb{R}^3$ that are not parallel must intersect.

   $\boxed{\text{False}}$

Quiz continues on the back

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2. Find the line of intersection of the planes $x + 2z = 4$ and $x - y - z = 1$.

Solution. The line of intersection is contained in both planes, so it has a direction vector orthogonal to each plane's normal vector. Therefore the direction vector is parallel to the cross product

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 2 \\ 1 & -1 & -1 \end{vmatrix} = \langle 2, 3, -1 \rangle.$$

To find a point on both planes, we can subtract the second plane equation from the first to get $y + 3z = 3$, then choose $y = 0$ so that $z = 1$ and (by substitution) $x = 2$. So one equation for the line of intersection is

$$\mathbf{r}(t) = \langle 2, 3, -1 \rangle t + \langle 2, 0, 1 \rangle.$$