Math 2551 Learning Target I1 Practice

I1: Double & Triple Integrals. I can set up double and triple integrals as iterated integrals over any region. I can sketch regions based on a given iterated integral.

Concept Checks

1. **T/F:**

$$\int_0^1 \int_0^1 1 \, dy \, dx > \int_0^1 \int_x^1 1 \, dy \, dx.$$

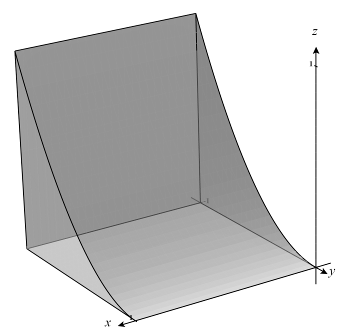
Open Response

1. Sketch the domain of integration corresponding to

$$\int_0^1 \int_{x^{2/3}}^1 x e^{y^4} dy \, dx.$$

Then change the order of integration and write a new iterated integral. Explain the simplification achieved by changing the order.

2. Let D be the solid in $\mathbb{R}^3$ which is bounded by the surface $z = y^2$ and the planes $z = 0, x = 0$, $x = 1, y = -1$, and $y = 0$, pictured to the right.



   (a) Write a triple integral in Cartesian coordinates for the volume of D using the order $dz\, dy\, dx$. Do not evaluate your integral.

   (b) Write a triple integral in Cartesian coordinates for the volume of D using the order $dx\, dy\, dz$. Do not evaluate your integral.

3. In this problem, you will compute the volume of the solid bounded above by $z = 3xy$, below by the xy-plane, and lying above the region R contained between the curves $2y = x^2$ and $2x = y^2$.

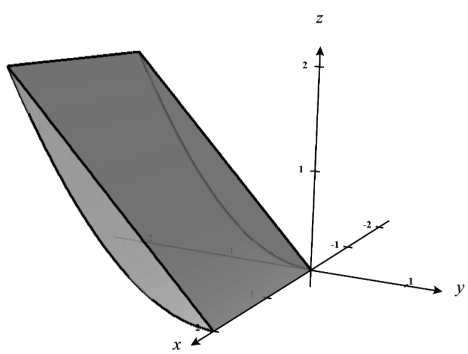
   (a) Sketch the region R. Label your axes and each curve.

(b) Is R horizontally simple, vertically simple, both, or neither?

(c) Write a double iterated integral to find the volume of the solid.

4. Consider the volume D bounded by the planes $x = 0, x = 2, z = -y$ and the surface $z = y^2/2$.

   (a) Write an integral for the volume of D using Cartesian coordinates in the order $dz\ dy\ dx$. Show your work, including a sketch of the shadow of the region. **Do not evaluate your integral**.



   (b) Write an integral for the volume of D using Cartesian coordinates in the order $dx\ dy\ dz$. Show your work, including a sketch of the shadow of the region. **Do not evaluate your integral**.

5. In this problem you will work with the region R bounded by $x = 1 + y^2, x = 1, y = 1$, and $y = 2$ to set up a double integral

$$I = \iint_R f(x, y)\ dA.$$

   (a) Sketch this region. Clearly label the region R and find all intersection points.

   (b) Determine whether the region is horizontally simple, vertically simple, bo

   (c) Based on your answer to (b), which order of integration will produce a simpler iterated integral?

   (d) Give an iterated integral expression (which may involve multiple integrals) for I using the order you chose in (c).

6. Sketch the domain of integration corresponding to

$$\int_0^1 \int_{x^{2/3}}^1 x \sin(y^4)\ dy\ dx.$$

   Then change the order of integration and evaluate. Explain the simplification achieved by changing the order.

7. In this problem, you will set up an integral for the area of the region R contained between the curves $y = 5/2$, $y = 4$, $x = 0$, and $y = \frac{1}{2}x^2 + 2$.

   (a) Sketch the region R. Label your axes and each curve.

   (b) Is R horizontally simple, vertically simple, both, or neither?

   (c) Write a double iterated integral to find the area of R.

8. Let $f(x, y, z) = 8xz$. Set up but do not evaluate an iterated integral in Cartesian coordinates using any order of integration for $\iiint_D f(x, y, z)\, dV$, where D is the region

$$y^2 \leq z \leq 8 - 2x^2 - y^2, \qquad x \geq 0$$

   A complete answer will include a sketch of the shadow of the region in an appropriate coordinate plane for your chosen direction of integration.

   Explain in 1-2 sentences why you chose the order of integration that you did.

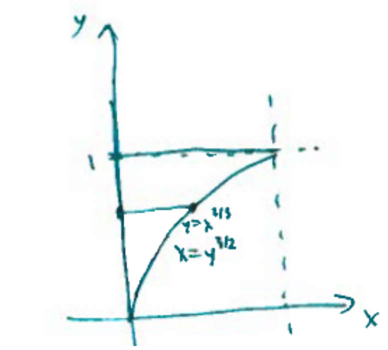
Math 2551 LT I1 Practice Answers

Concept Checks

1. True.

Open Response

1. We have $x^{2/3} \leq y \leq 1$ and $0 \leq x \leq 1$, so the domain of integration is:



Reversing the order of integration therefore yields

$$\int_0^1 \int_0^{y^{3/2}} xe^{y^4} \, dx \, dy.$$

This allows a u-substitution for the second integration step which will make the problem much simpler.

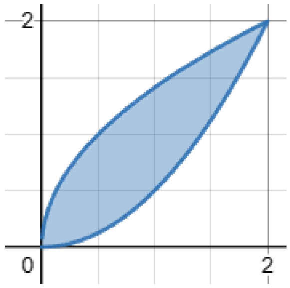
2. (a)

$$V = \int_0^1 \int_{-1}^0 \int_0^{y^2} dz \, dy \, dx$$

(b)

$$V = \int_0^1 \int_{-1}^{-\sqrt{z}} \int_0^1 dx \, dy \, dz$$

3. In this problem, you will compute the volume of the solid bounded above by $z = 3xy$, below by the xy-plane, and lying above the region R contained between the curves $2y = x^2$ and $2x = y^2$.

(a) 

(b) R is both horizontally and vertically simple.

(c) We can choose either order of integration, which results in one of the following two integrals.

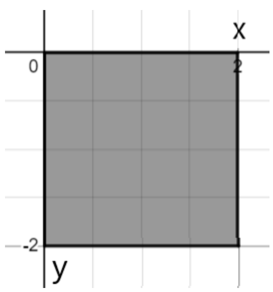
$$\int_0^2 \int_{x^2/2}^{\sqrt{2x}} 3xy\, dy\, dx$$

or

$$\int_0^2 \int_{y^2/2}^{\sqrt{2y}} 3xy\, dx\, dy.$$

4. Consider the volume D bounded by the planes $x = 0, x = 2, z = -y$ and the surface $z = y^2/2$.

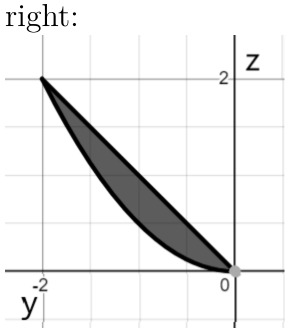
(a) The shadow of this region in the xy-plane is a square:



We have

$$V = \int_0^2 \int_{-2}^{0} \int_{y^2/2}^{-y} dz\, dy\, dx.$$

(b) The shadow of this region in the yz-plane is shown to the right:

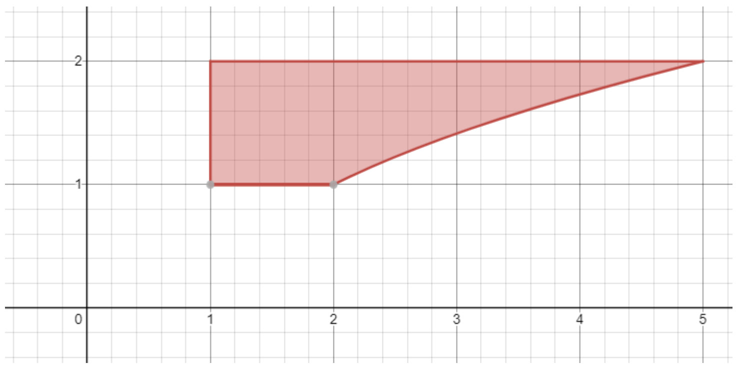


We have

$$V = \int_0^2 \int_{-\sqrt{2z}}^{-z} \int_0^2 \; dx\; dy\; dz.$$

5. In this problem you will work with the region R bounded by $x = 1 + y^2, x = 1, y = 1$, and $y = 2$ to set up a double integral

$$I = \iint_R f(x, y)\; dA.$$

(a) 

The horizontal lines intersect the parabola at $(2, 1)$ and $(5, 2)$.

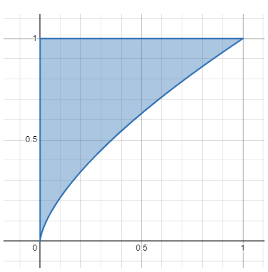
(b) The region is only horizontally simple: arrows drawn from left to right always enter the region along the vertical line and leave along the parabola, but arrows drawn from bottom to top change behavior at $x = 2$ where they shift from entering along the horizontal line to entering along the parabola.

(c) Since the region is only horizontally simple, we should use the $dx\; dy$ order of integration.

(d)

$$I = \int_1^2 \int_1^{1+y^2} f(x, y)\; dx\; dy.$$

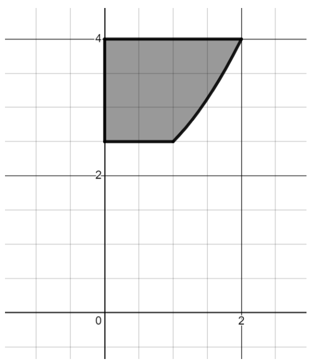
6. A sketch of the region is below:



We have

$$\begin{aligned}\int_0^1 \int_{x^{2/3}}^1 x\sin(y^4)\ dy\ dx &= \int_0^1 \int_0^{y^{3/2}} x\sin(y^4)\ dx\ dy \\ &= \int_0^1 \frac{1}{2}x^2\sin(y^4)|_{x=0}^{x=y^{3/2}}\ dy \\ &= \int_0^1 \frac{1}{2}y^3\sin(y^4)\ dy \\ &= -\frac{1}{8}\cos(y^4)|_0^1 \\ &= \frac{1}{8}(1-\cos(1))\end{aligned}$$

This simplifies the integral by allowing a u-substitution to deal with the $\sin(y^4)$.

7. (a) 

   (b) R is only horizontally simple.

   (c) The simpler order of integration is $dx\ dy$, resulting in

$$\int_{5/2}^4 \int_0^{\sqrt{2y-4}} 1\ dx\ dy.$$

   The other order requires splitting the integral at $x = 1$:

$$\int_0^1 \int_{5/2}^4 1\ dy\ dx + \int_1^2 \int_{\frac{1}{2}x^2+2}^4 1\ dy\ dx.$$

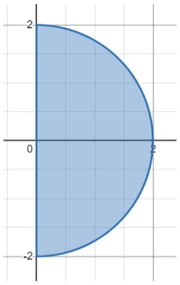
8. Use the order $dz\ dx\ dy$. The shadow of this region in the xy-plane is given by

$$y^2 \le 8 - 2x^2 - y^2, \qquad x \ge 0.$$

Simplifying the first inequality yields $2x^2 + 2y^2 \le 8$, so our shadow is

$$x^2 + y^2 \le 4, x \ge 0,$$

which is the right half of a circle of radius 2 centered at the origin.



Therefore our triple iterated integral is

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{y^2}^{8-2x^2-y^2} 8xz\;dz\;dx\;dy.$$

We chose this order because both z bounds are given already, so starting with z makes sense. Then we could have chosen either order for x and y, but the bounds on x are slightly nicer (no negative square root) so we use $dx\;dy$.