Math 2551 Learning Target G5 Practice

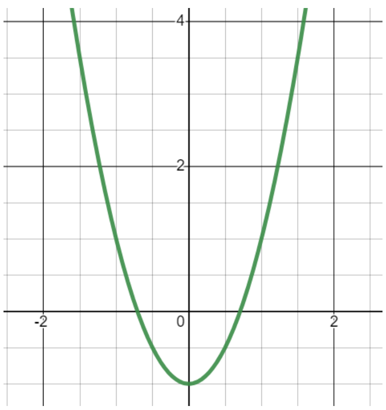
G5: Parameterization. I can find parametric equations for common curves, such as line segments, graphs of functions of one variable, circles, and ellipses. I can match given parametric equations to Cartesian equations and graphs. I can parameterize common surfaces, such as planes, quadric surfaces, and functions of two variables.

Concept Checks

1. **T/F:** There is exactly one possible parameterization for any curve in space.
2. **T/F:** The graphs of $f(x, y) = x^2 + y^2$ and $g(x, y) = 1 - x^2 - y^2$ intersect in a circle.

Open Response

3. Suppose $\mathbf{r}(t) = e^t\mathbf{i} + (2e^{2t} - 1)\mathbf{j}$ for $-\infty < t < \infty$. Below is the graph of the curve with equation $y = 2x^2 - 1$. Does $\mathbf{r}(t)$ parameterize this curve? Explain why or why not.



4. In this problem, you will work with the curve C that consists of the quarter of the circle $x^2 + y^2 = 9$ from $(3, 0)$ to $(0, 3)$, followed by the straight line segment back to $(3, 0)$. Give a parameterization for this curve. Be sure to give a domain.

   Hint: you may need two vector-valued functions.

5. The sphere $x^2 + y^2 + z^2 = 6$ meets the paraboloid $z = x^2 + y^2$ in a circle. Give a parameterization of this circle. Be sure to give a domain.
6. Consider the surface S which is the part of the plane $x + y + z = 1$ inside the cylinder $x^2 + y^2 = 1$. Give a parameterization of the curve C which is the boundary of S. Be sure to give a domain.

7. Find a vector function that represents the curve of intersection of the paraboloid $z = 4x^2 + 4y^2$ and the cylinder $y = 2x^2$. Be sure to give a domain.

8. Find a parameterization of the curve that lies in the plane $x + y - 2z = 4$ above the points with $x = 2 + 3y^3$ for $-1 \le y \le 1$. Be sure to give a domain.

9. The plane $x + 2y - z = 1$ meets the cylinder $y^2 + z^2 = 4$ in an ellipse. Parameterize this ellipse. Be sure to give a domain.

10. The vector valued function

$$\mathbf{r}(t) = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + 2\cos(t)\left(\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}\right) + 2\sin(t)\left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

describes the motion of an object moving in a circle of radius 2 centered at the point $(2, -1, 4)$.

Find the plane $2x + 2y - 3z = d$ that contains this circle.

Math 2551 LT G5 Practice Answers

Concept Checks

1. False
2. True

Open Response

3. No, because although $\mathbf{r}(t)$ does satisfy $y(t) = 2x(t)^2 - 1$, the domains do not match. $\mathbf{r}(t)$ only gives points with $x(t) > 0$, while the domain of $y = 2x^2 - 1$ is all real values of x.

4. The quarter circle is parameterized by $\mathbf{r}_1(t) = \langle 3\cos(t), 3\sin(t)\rangle$ for $0 \le t \le \pi/2$ and the straight line segment by $\mathbf{r}_2(t) = \langle 3t, 3-3t\rangle$ for $0 \le t \le 1$.

5. Since $x^2 + y^2 = z$, we have $z + z^2 = 6$ along the intersection of these surfaces. So $z^2 + z - 6 = (z+3)(z-2) = 0$. Since $z = x^2 + y^2 \ge 0$, the only solution is $z = 2$. Hence the intersection is the circle $x^2 + y^2 = 2$ in the plane $z = 2$, which can be parameterized as

$$\mathbf{r}(t) = \langle \sqrt{2}\cos(t), \sqrt{2}\sin(t), 2\rangle, \qquad 0 \le t \le 2\pi.$$

6. The boundary of S is the intersection of these surfaces, hence the components of $\mathbf{r}(t)$ must satisfy both surface equations. From the cylinder equation we know that we can take $x(t) = \cos(t), y(t) = \sin(t)$. So we must have (from the plane equation) $z(t) = 1 - \cos(t) - \sin(t)$. Finally, our domain is $0 \le t \le 2\pi$.

7. x is independent in both equations, so we let $x(t) = t$. Then from the cylinder we have $y(t) = 2t^2$ and so the paraboloid forces $z(t) = 4t^2 + 16t^4$. So a parameterization is

$$\mathbf{r}(t) = \langle t, 2t^2, 4t^2 + 16t^4\rangle, \quad t \in \mathbb{R}.$$

8. Since this curve lies above the curve $x = 2 + 3y^3, -1 \le y \le 1$ and lies in the plane $z = \frac{x}{2} + \frac{y}{2} - 2$ we have

$$\mathbf{r}(t) = \langle 2 + 3t^3, t, -1 + \frac{t}{2} + \frac{3t^3}{2}\rangle, -1 \le t \le 1.$$

9. We can parameterize this intersection curve by parameterizing the circular cross-section of the cylinder and then using the plane equation to get the final coordinate.

   From the cylinder
$$y(t) = 2\cos(t) \qquad z(t) = 2\sin(t)$$
   and then the plane gives $x = 1 - 2y + z$ so the final parameterization is
$$\mathbf{r}(t) = \langle 1 - 4\cos(t) + 2\sin(t), 2\cos(t), 2\sin(t)\rangle, 0 \le t \le 2\pi.$$

10. We simply substitute $x(t), y(t)$, and $z(t)$ into the equation of the plane to find d:

$$\begin{aligned} 2(2 + 2\cos(t)/\sqrt{2} + 2\sin(t)/3) + 2(-1 - 2\cos(t)/\sqrt{2} + 4\sin(t)/3) - 3(4 + 4\sin(t)/3) &= d \\ 4 + 4\cos(t)/\sqrt{2} + 4\sin(t)/3 - 2 - 4\cos(t)/\sqrt{2} + 8\sin(t)/3 - 12 - 12\sin(t)/3 &= d \\ -10 &= d. \end{aligned}$$

   So the plane is $2x + 2y - 3z = -10$.