

Math 2551 Learning Target G4 Practice

G4: Surfaces. I can identify standard quadric surfaces including: spheres, ellipsoids, elliptic paraboloids, hyperboloids, cones, and hyperbolic paraboloids. I can match graphs of functions of two variables to their equations and contour plots and determine their domains and ranges.

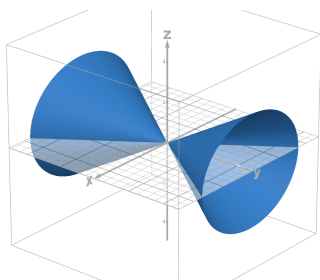
Concept Checks

1. **T/F:** The sphere $x^2 + (y - 2)^2 + (z + 1)^2 = 6$ has center $(0, -2, 1)$.
2. **T/F:** The domain of the function $f(x, y) = \sqrt{x - 2} + y$ is the interval $[2, \infty)$.
3. **T/F:** All surfaces in $\mathbb{R}^3$ are quadric surfaces.
4. **T/F:** Level surfaces of the function $f(x, y, z) = \sqrt{x^2 + y^2}$ are:
 - (a) Circles centered at the origin
 - (b) Spheres centered at the origin
 - (c) Cylinders centered around the z -axis
 - (d) Upper hemispheres centered at the origin
 - (e) None of the above
5. **T/F:** The contour map for a hill and the contour map for a valley appear the same if the contours are not labeled.
6. Which of the following is true of the quadric surface $z = x^2 + 2y^2$?
 - (a) It is a plane, because all of its cross-sections in the $x = k$, $y = k$, and $z = k$ planes are straight lines.
 - (b) It is a sphere, because all of its cross-sections in the $x = k$, $y = k$, and $z = k$ planes are circles.
 - (c) It is an elliptical paraboloid, because its cross-sections are ellipses in the $z = k$ planes and parabolas in the $x = k$ and $y = k$ planes.
 - (d) It is a hyperbolic paraboloid, because its cross-sections are hyperbolas in the $z = k$ planes and parabolas in the $x = k$ and $y = k$ planes.

- (e) It is a cone, because its cross-sections are circles in the $z = k$ planes and straight lines in the $x = k$ and $y = k$ planes.
7. **T/F:** Any surface that is a graph of a function of two variables $z = f(x, y)$ can be thought of as a level surface of a function of 3 variables.
8. Classify the quadric surface shown below.

<i>A</i>	Ellipsoid
<i>B</i>	Elliptical Paraboloid
<i>C</i>	Elliptical Cone
<i>D</i>	Hyperboloid of one Sheet

<i>E</i>	Hyperboloid of two Sheets
<i>F</i>	Hyperbolic Paraboloid
<i>G</i>	Plane
<i>H</i>	Elliptical Cylinder

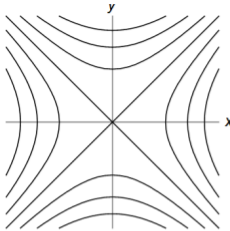


9. **T/F:** The surface in $\mathbb{R}^3$ consisting of the points which solve the equation $x^3 + y^3 - z = 27$ is an example of a quadric surface.
10. **T/F:** The point $(2, -1, 3)$ lies on the graph of the sphere $(x-2)^2 + (y+1)^2 + (z-3)^2 = 25$.
11. **T/F:** If the graph of $z = f(x, y)$ is a plane, then each cross section is a line.
12. **T/F:** The contour lines of the graph of $f(x, y) = 2x + 3y$ are parallel lines.
13. **T/F:** The cross section of the function $f(x, y) = x + y^2$ in the plane $y = 1$ is a line.
14. **T/F:** The contours of the graph of $f(x, y) = y^2 + (x - 2)^2$ are either circles or a single point.
15. **T/F:** If all the contours for $f(x, y)$ are parallel lines, then the graph of f is a plane.
16. **T/F:** The level surfaces of the function $f(x, y, z) = x^2 + y^2 + z^2$ are cylinders with axis along the y -axis.

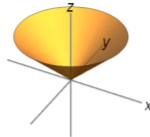
Open Response

17. (a) Find and sketch the domain of the function $f(x, y) = \sqrt{4 - x^2 - y^2}$. Show your work and label your axes clearly.
- (b) Match the contour plots and graphs below with the given functions of two variables.

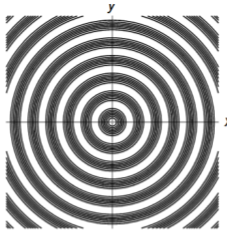
Function	Graph	Contour Plot
$f(x,y) = \cos\left(\sqrt{x^2+y^2}\right)$	_____	_____
$g(x,y) = x^2 - y^2$	_____	_____
$h(x,y) = x^2 + 2y^2$	_____	_____



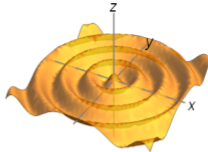
A



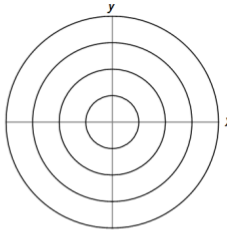
B



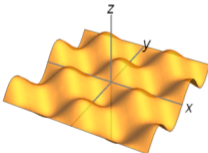
C



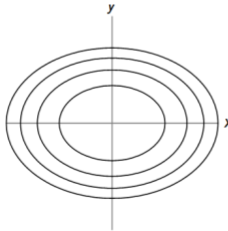
D



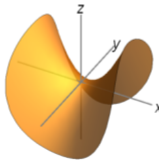
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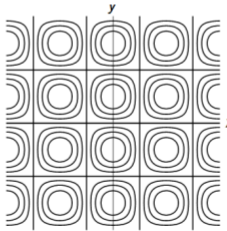
F



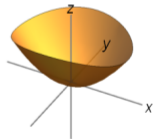
G



H



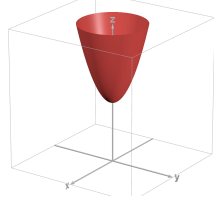
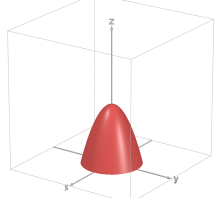
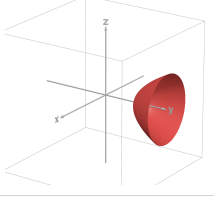
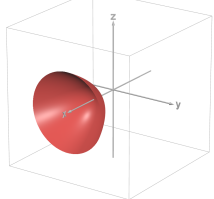
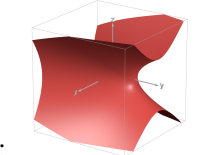
I



J

18. (a) In this part, you will consider the surface with equation $y - x^2 - z^2 = 1$. Fill in each blank with the appropriate letter.

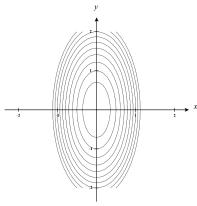
- i. The cross sections in the planes $x = k$ are _____
- ii. The cross sections in the planes $y = k$ are _____
- iii. The cross sections in the planes $z = k$ are _____
- iv. The quadric surface is a _____
- v. The surface looks most like graph _____

Cross Section Choices	Quadric Surface Choices	Graph Choices
A: Ellipses/Circles	E: Ellipsoid	K: 
B: Lines	F: Elliptical Paraboloid	L: 
C: Parabolas	G: Hyperbolic Paraboloid	M: 
D: Hyperbolas	H: Hyperboloid of 1 or 2 Sheets	N: 
	I: Cone	O: 

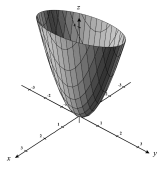
- (b) Find and sketch the domain of the function $f(x, y) = \frac{\sqrt{4 - (x - 2)^2 - y^2}}{x^2 e^y}$. Be sure to clearly indicate which points on the boundary are included or excluded.

19. Match the contour plots and graphs below with the given functions of two variables.

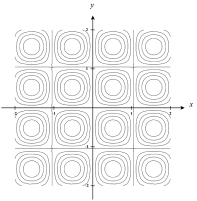
Function	Graph	Contour Plot
$z = \sin(3x) \sin(3y)$	_____	_____
$z = 2e^{-(x^2+y^2)}$	_____	_____
$z = 4x^2 + y^2$	_____	_____



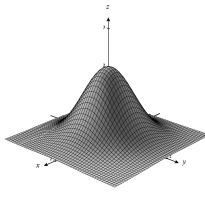
(A)



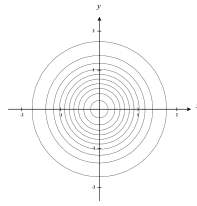
(B)



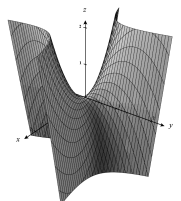
(C)



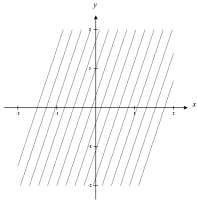
(D)



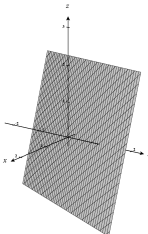
(E)



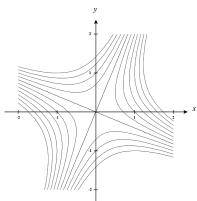
(F)



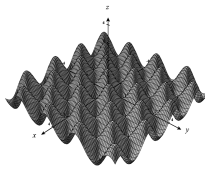
(G)



(H)



(I)

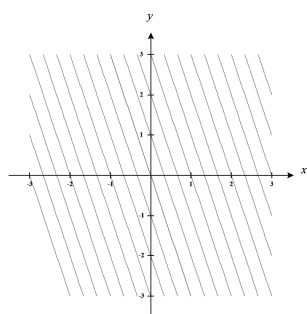


(J)

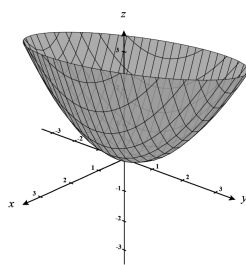
20. Let $f(x, y) = \frac{\ln(x - y)}{\sqrt{4 - x^2 - y^2}}$. Graph the domain of f . Be sure to show all your work in finding the domain. Indicate whether or not each part of the boundary of the domain is included.

21. Match each function or quadric surface to its graph and contour plot.

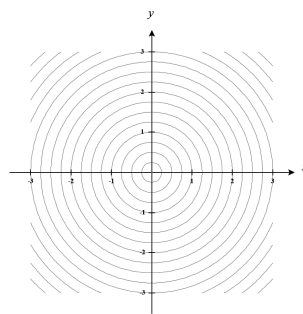
Function	Graph	Contour Plot
$z = 3x + y$	_____	_____
$z = \sqrt{x^2 + y^2}$	_____	_____
$z = x^2 - y^2$	_____	_____
$z = x^2 + y^2/4$	_____	_____



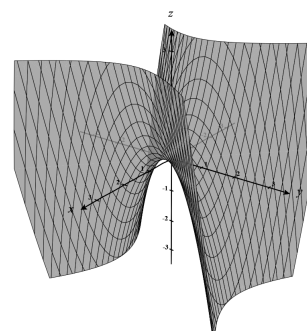
(A)



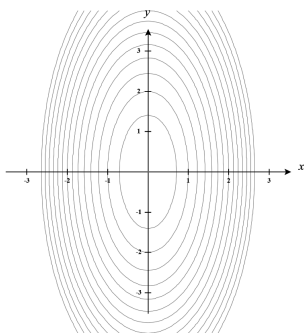
(B)



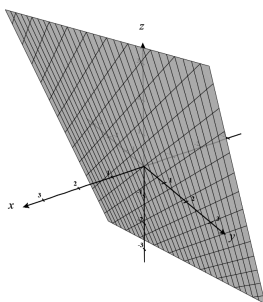
(C)



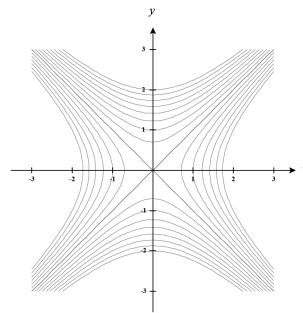
(D)



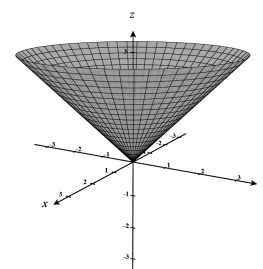
(E)



(F)



(G)



(H)

22. Find and sketch the domain of the function

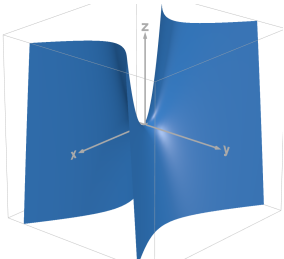
$$f(x, y) = \frac{\sqrt{4 - x^2}}{x^2 + y^2 - 1}.$$

Clearly indicate points that are or are not included in the domain in your sketch.

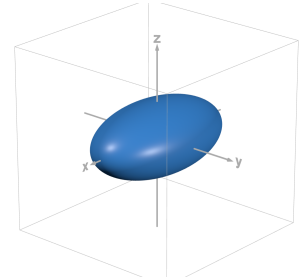
23. Classify each quadric surface shown below by writing the appropriate letter into the provided box.

<i>A</i>	Ellipsoid
<i>B</i>	Elliptical Paraboloid
<i>C</i>	Elliptical Cone
<i>D</i>	Hyperboloid of one Sheet

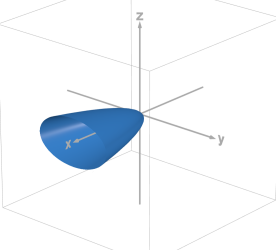
<i>E</i>	Hyperboloid of two Sheets
<i>F</i>	Hyperbolic Paraboloid
<i>G</i>	Plane
<i>H</i>	Elliptical Cylinder



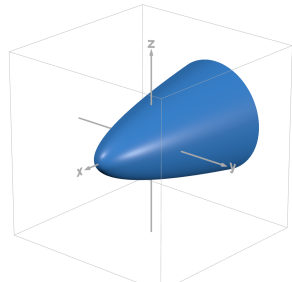
Answer:



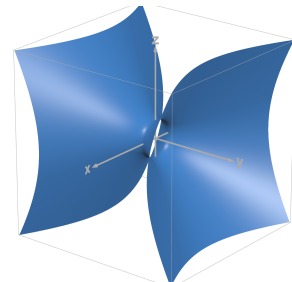
Answer:



Answer:

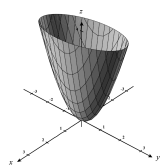


Answer:

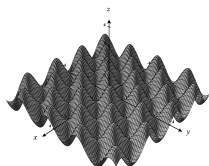


Answer:

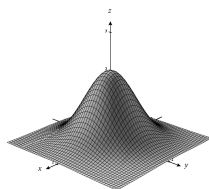
24. Match each provided graph with its contour plot (A)-(E). Each plot must be used exactly once.



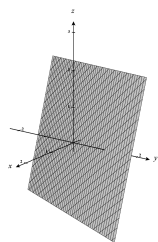
Answer:



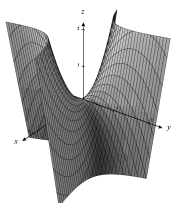
Answer:



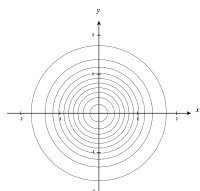
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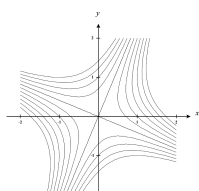
Answer:



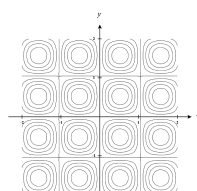
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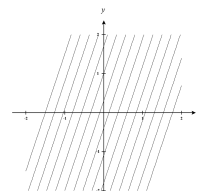
(A)



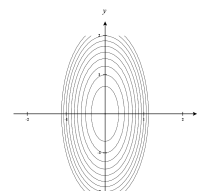
(B)



(C)



(D)



(E)

25. Find and sketch the domain of the function

$$f(x, y) = \frac{\sqrt{9 - (x - 1)^2 - (y - 1)^2}}{x^2 e^y}.$$

Be sure to clearly indicate which points on the boundary are included or excluded.

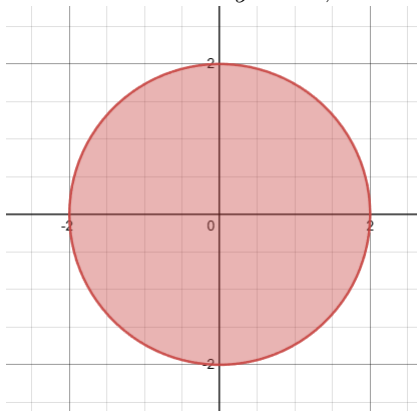
Math 2551 LT G4 Practice Answers

Concept Checks

1. False.
2. False.
3. False.
4. (c)
5. True.
6. (c)
7. True
8. C
9. False
10. False.
11. True
12. True
13. True
14. True
15. False
16. False

Open Response

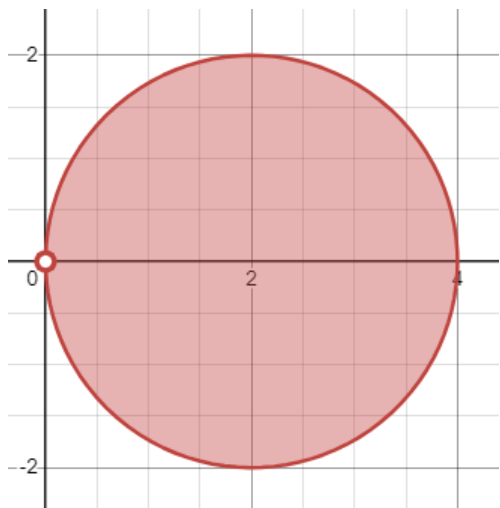
17. (a) We need $4 - x^2 - y^2 \geq 0$, so the domain is all (x, y) inside or on the circle $x^2 + y^2 = 4$.



- (b) f : Graph D, Contour Plot C.
 g : Graph H, Contour Plot A.
 h : Graph J, Contour Plot G.
18. (a) In this part, you will consider the surface with equation $y - x^2 - z^2 = 1$. Fill in each blank with the appropriate letter.
- i. The cross sections in the planes $x = k$ are **C: Parabolas**
 - ii. The cross sections in the planes $y = k$ are **A: Ellipses/Circles**
 - iii. The cross sections in the planes $z = k$ are **C: Parabolas**
 - iv. The quadric surface is an **F: Elliptical Paraboloid**
 - v. The surface looks most like graph **M**
- (b) The domain is all points (x, y) such that

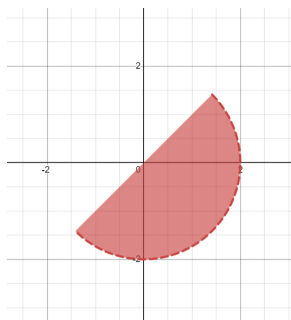
$$4 - (x - 2)^2 - y^2 \geq 0 \quad \text{and} \quad x^2 e^y \neq 0.$$

The former gives the disk $(x - 2)^2 + y^2 \leq 4$ of radius 2 centered at $(2, 0)$, and the latter excludes the line $x = 0$. The only point in this disk with $x = 0$ is the origin, so we get the sketch below.



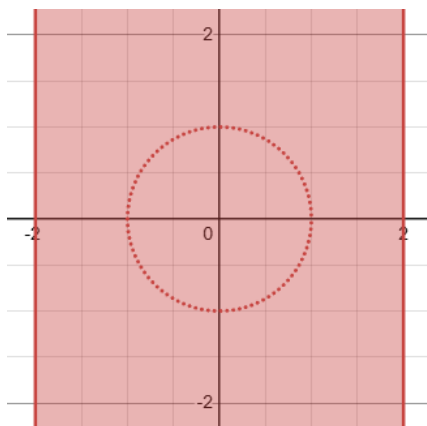
	Function	Graph	Contour Plot
19.	$z = \sin(3x) \sin(3y)$	<u>(J)</u>	<u>(C)</u>
	$z = 2e^{-(x^2+y^2)}$	<u>(D)</u>	<u>(E)</u>
	$z = 4x^2 + y^2$	<u>(B)</u>	<u>(A)</u>

20. We need $x - y > 0$ and $4 - x^2 - y^2 > 0$ so that all functions are well-defined and we do not divide by zero. Thus the domain is all points (x, y) which are both to the right/below $y = x$ and inside the circle $x^2 + y^2 = 4$. No boundary points are included.



	Function	Graph	Contour Plot
21.	$z = 3x + y$	<u>(F)</u>	<u>(A)</u>
	$z = \sqrt{x^2 + y^2}$	<u>(H)</u>	<u>(C)</u>
	$z = x^2 - y^2$	<u>(D)</u>	<u>(G)</u>
	$z = x^2 + y^2/4$	<u>(B)</u>	<u>(E)</u>

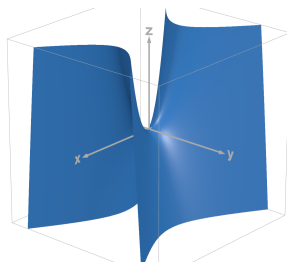
22. From the square root, we have $4 - x^2 \geq 0$, so $|x| \leq 2$. From the denominator, we have $x^2 + y^2 \neq 1$. So the domain is all points (x, y) with $-2 \leq x \leq 2$ except those on the unit circle $x^2 + y^2 = 1$. This is sketched below.



23. Classify each quadric surface shown below by writing the appropriate letter into the provided box.

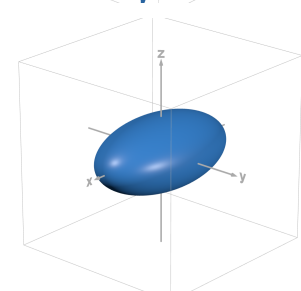
<i>A</i>	Ellipsoid
<i>B</i>	Elliptical Paraboloid
<i>C</i>	Elliptical Cone
<i>D</i>	Hyperboloid of one Sheet

<i>E</i>	Hyperboloid of two Sheets
<i>F</i>	Hyperbolic Paraboloid
<i>G</i>	Plane
<i>H</i>	Elliptical Cylinder



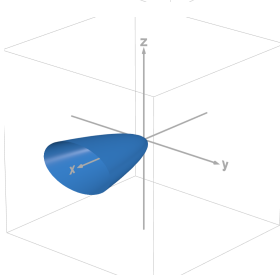
Answer:

F



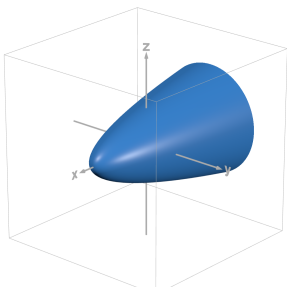
Answer:

A

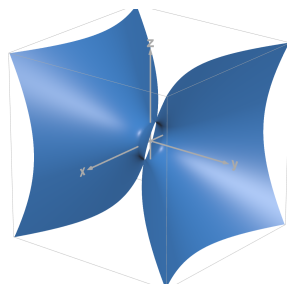


Answer:

B



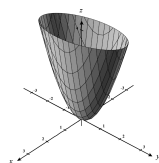
Answer:

 B 

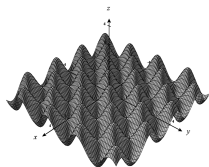
Answer:

 D

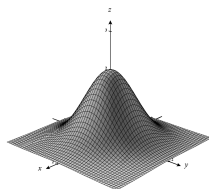
24. Match each provided graph with its contour plot (A)-(E). Each plot must be used exactly once.



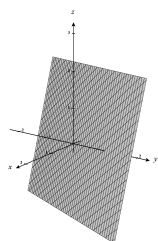
Answer: 30E



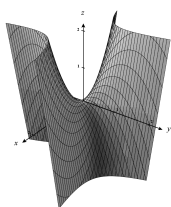
Answer: 30C



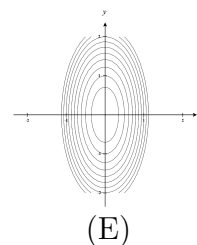
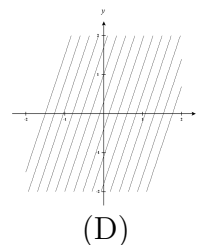
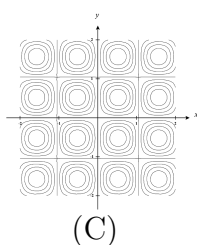
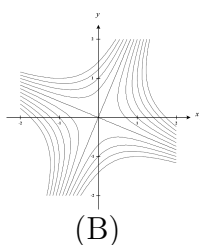
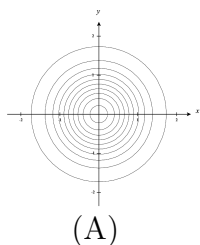
Answer: 30A



Answer: 30D



Answer: 30B



25. From the numerator we have that $9 - (x - 1)^2 - (y - 1)^2 \geq 0$ and from the denominator we have $x \neq 0$. Combining these, the domain is the disk of radius 3 centered at $(1, 1)$, including its bounding circle, but excluding the y -axis.

