Daily Announcements & Reminders:

Goals for Today: Section 14.1

- Introduce and sketch traces and contours of functions of two variables
- Find level surfaces of functions of three variables
- Graph functions of two variables

Last time, we discussed the domains of the functions $f(x,y) = x^2 + y^2, g(x,y) = \ln(x+y)$, and $h(x,y) = \dfrac{1}{\sqrt{x+y}}$.

Definition 31. If f is a function of two variables with domain D, then the graph of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

Here are the graphs of the three functions above.

Example 32. Suppose a small hill has height $h(x, y) = 4 - \frac{1}{4}x^2 - \frac{1}{4}y^2$ m at each point (x, y). How could we draw a picture that represents the hill in 2D?

In 3D, it looks like this.

Definition 33. The ____________ (also called ____________________) of a function f of two variables are the curves with equations ________________, where k is a constant (in the range of f). A plot of ____________ for various values of z is a ____________________(or ____________________________).

Some common examples of these are:

-
-
-

Example 34. Create a contour diagram of $f(x, y) = x^2 - y^2$

Definition 35. The ______________ of a surface are the curves of ____________________ of the surface with planes parallel to the ________________________.

Example 36. Use the traces and contours of $z = f(x, y) = 4 - 2x - y^2$ to sketch the portion of its graph in the first octant.

Let's check our work: https://tinyurl.com/math2551-2var-graph

Definition 37. A ________________________________ is a rule that assigns to each __________ of real numbers (x, y, z) in a set D a ________________ denoted by $f(x, y, z)$.

$$f : D \to \mathbb{R}, \text{ where } D \subseteq \mathbb{R}^3$$

We can still think about the domain and range of these functions. Instead of level curves, we get level surfaces.

Example 38. Describe the domain of the function $f(x, y, z) = \dfrac{1}{4 - x^2 - y^2 - z^2}$.

Example 39. Describe the level surfaces of the function $g(x, y, z) = 2x^2 + y^2 + z^2$.

Section 14.2 Limits & Continuity

Definition 40. What is a limit of a function of two variables?

DEFINITION We say that a function $f(x, y)$ approaches the **limit** L as (x, y) approaches (x_0, y_0), and write

$$\lim_{(x, y) \to (x_0, y_0)} f(x, y) = L$$

if, for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all (x, y) in the domain of f,

$$|f(x, y) - L| < \epsilon \quad \text{whenever} \quad 0 < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta.$$

We won't use this definition much: the big idea is that $\lim_{(x,y) \to (x_0,y_0)} f(x, y) = L$ if and only if $f(x, y)$ ______________________________ regardless of how we approach (x_0, y_0).