Daily Announcements & Reminders:

Goals for Today: Sections 13.3-13.4

- Define, interpret, and compute the curvature of a curve
- Compute the unit tangent and principal unit normal vectors of a curve
- Give examples of functions of multiple variables
- Find the domain of functions of two variables
- Graph functions of two variables

13.4 - Curvature, Tangents, Normals

The next idea we are going to explore is the curvature of a curve in space along with two vectors that orient the curve.

First, we need the **unit tangent vector**, denoted $\mathbf{T}$:

- In terms of an arc-length parameter s: ________________

- In terms of any parameter t: ________________

This lets us define the **curvature**, $\kappa(s) =$ ________________

Example 23. Last class we found an arc length parameterization of the circle of radius 4 centered at $(0,0)$ in $\mathbb{R}^2$:

$$\mathbf{r}(s) = \left\langle 4\cos\left(\frac{s}{4}\right), 4\sin\left(\frac{s}{4}\right)\right\rangle, \qquad 0 \le s \le 8\pi.$$

Use this to find $\mathbf{T}(s)$ and $\kappa(s)$.

Question: In which direction is $\mathbf{T}$ changing?

This is the direction of the **principal unit normal**, $\mathbf{N}(s) =$________________

We said last time that it is often hard to find arc length parameterizations, so what do we do if we have a generic parameterization $\mathbf{r}(t)$?

- $\mathbf{T}(t) =$ ________________
- $\mathbf{N}(t) =$ ________________
- $\kappa(t) =$ ________________ or ________________________

Example 24. Find $\mathbf{T}, \mathbf{N}, \kappa$ for the helix $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), t-1 \rangle$.

14.1 Functions of Multiple Variables

Definition 25. A ________________________________ is a rule that assigns to each __________ of real numbers (x, y) in a set D a ________________ denoted by $f(x, y)$.

$$f : D \to \mathbb{R}, \text{ where } D \subseteq \mathbb{R}^2$$

Example 26. Three examples are

$$f(x, y) = x^2 + y^2, \qquad g(x, y) = \ln(x + y), \qquad h(x, y) = \frac{1}{\sqrt{x + y}}.$$

Example 27. Find the largest possible domains of f, g, and h.

Definition 28. If f is a function of two variables with domain D, then the graph of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

Here are the graphs of the three functions above.

Example 29. Suppose a small hill has height $h(x, y) = 4 - \frac{1}{4}x^2 - \frac{1}{4}y^2$ m at each point (x, y). How could we draw a picture that represents the hill in 2D?

In 3D, it looks like this.

Definition 30. The ____________ (also called ____________________) of a function f of two variables are the curves with equations ________________, where k is a constant (in the range of f). A plot of ____________ for various values of z is a ____________________(or ____________________________).

Some common examples of these are:

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