Daily Announcements & Reminders:

Goals for Today: Sections 13.1-13.3

- Compute limits, derivatives, and tangent lines for vector-valued functions
- Compute integrals of vector-valued functions and solve initial value problems
- Compute arc lengths of curves using parameterizations
- Introduce the idea of an arc-length parameterization

Continuing from last time with <u>derivatives</u>:

Example 16. If $\mathbf{r}(t) = \langle 2t - \frac{1}{2}t^2 + 1, t - 1 \rangle$, find $\mathbf{r}'(t)$.

Interpretation: If $\mathbf{r}(t)$ gives the position of an object at time t, then

- $\mathbf{r}'(t)$ gives ________________________________

- $|\mathbf{r}'(t)|$ gives ________________________________

- $\mathbf{r}''(t)$ gives ________________________________

Let's see this graphically

Example 17. Find an equation of the tangent line to $\mathbf{r}(t) = \langle 2t - \frac{1}{2}t^2 + 1, t - 1 \rangle$ at time $t = 2$.

And with integrals:

Example 18. Find $\int_0^1 \langle t, e^{2t}, \sec^2(t)\rangle \; dt$.

At this point we can solve initial-value problems like those we did in single-variable calculus:

Example 19. Wallace is testing a rocket to fly to the moon, but he forgot to include instruments to record his position during the flight. He knows that his velocity during the flight was given by

$$\mathbf{v}(t) = \langle -200\sin(2t), 200\cos(t), 400 - \frac{400}{1+t}\rangle \; m/s.$$

If he also knows that he started at the point $\mathbf{r}(0) = \langle 0, 0, 0\rangle$, use calculus to reconstruct his flight path.

13.3 Arc length of curves

We have discussed motion in space using by equations like $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$.

Our next goal is to be able to measure distance traveled or arc length.

Motivating problem: Suppose the position of a fly at time t is

$$\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t) \rangle,$$

where $0 \leq t \leq 2\pi$.

a) Sketch the graph of $\mathbf{r}(t)$. What shape is this?

b) How far does the fly travel between $t = 0$ and $t = \pi$?

c) What is the speed $|\mathbf{v}(t)|$ of the fly at time t?

d) Compute the integral $\displaystyle\int_0^{\pi} |\mathbf{v}(t)| \, dt$. What do you notice?

Definition 20. We say that the **arc length** of a smooth curve $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ from ____________ to ____________ that is traced out exactly once is

$$L = \underline{\hspace{6cm}}$$

Example 21. Set up an integral for the arc length of the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ from the point $(1, 1, 1)$ to the point $(2, 4, 8)$.

Sometimes, we care about the distance traveled from a fixed starting time t_0 to an arbitrary time t, which is given by the **arc length function**.

$$s(t) = \underline{\hspace{6cm}}$$

We can use this function to produce parameterizations of curves where the parameter s measures distance along the curve: the points where $s = 0$ and $s = 1$ would be exactly 1 unit of distance apart.

Example 22. Find an arc length parameterization of the circle of radius 4 about the origin in $\mathbb{R}^2$, $\mathbf{r}(t) = \langle 4\cos(t), 4\sin(t)\rangle, 0 \leq t \leq 2\pi$.