Daily Announcements & Reminders:

Goals for Today: Sections 12.6, 13.1

- Sketch quadric surfaces in $\mathbb{R}^3$
- Introduce vector-valued functions
- Plot vector-valued functions and construct them from a graph
- Compute limits, derivatives, and tangent lines for vector-valued functions

Section 12.6 Quadric Surfaces

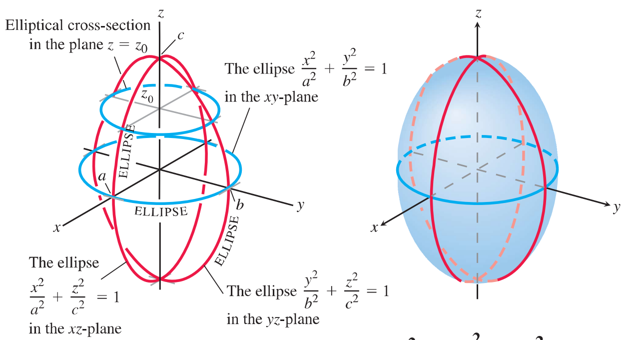
Definition 11. A **quadric surface** in $\mathbb{R}^3$ is the set of points that solve a quadratic equation in x, y, and z.

You know several examples already:

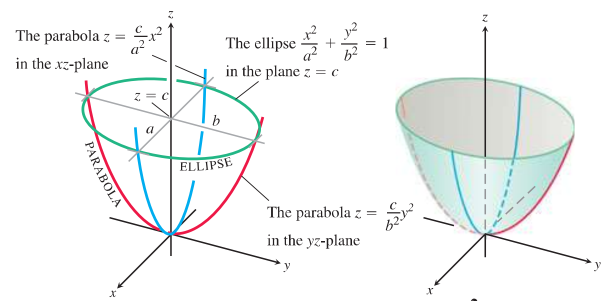
The most useful technique for recognizing and working with quadric surfaces is to examine their cross-sections.

Example 12. Use cross-sections to sketch and identify the quadric surface $x = z^2 + y^2$.

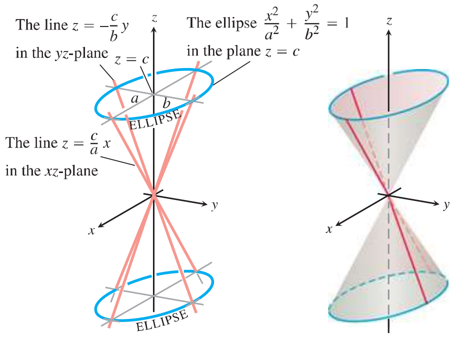
TABLE 12.1 Graphs of Quadric Surfaces



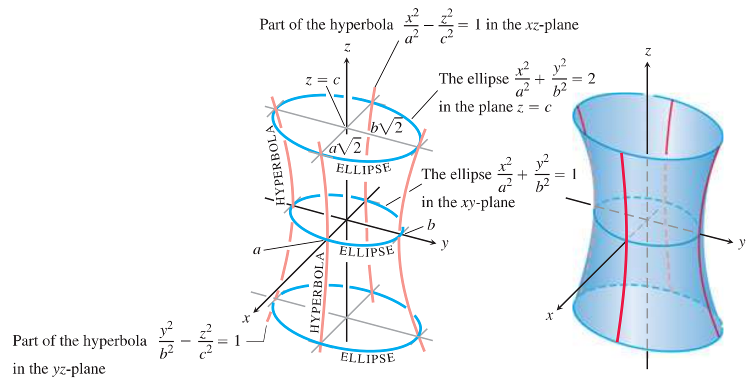
ELLIPSOID $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



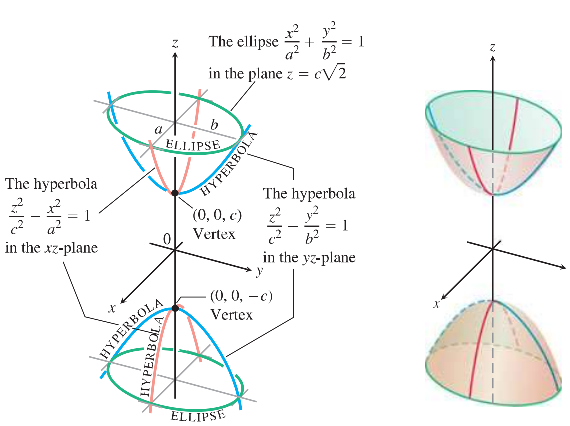
ELLIPTICAL PARABOLOID $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$



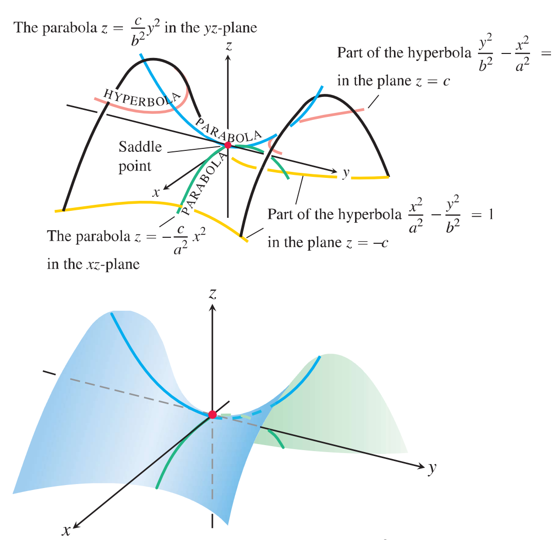
ELLIPTICAL CONE $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$



HYPERBOLOID OF ONE SHEET $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



HYPERBOLOID OF TWO SHEETS $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



HYPERBOLIC PARABOLOID $\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}, c > 0$

Section 13.1 Curves in Space & Their Tangents

The description we gave of a line last week generalizes to produce other one-dimensional graphs in $\mathbb{R}^2$ and $\mathbb{R}^3$ as well. We said that a function $\mathbf{r} : \mathbb{R} \to \mathbb{R}^3$ with $\mathbf{r}(t) = \mathbf{v}t + \mathbf{r}_0$ produces a straight line when graphed.

This is an example of a **vector-valued function**: its input is a real number t and its output is a vector. We graph a vector-valued function by plotting all of the terminal points of its output vectors, placing their initial points at the origin.

You have seen several examples already:

Given a fixed curve C in space, producing a vector-valued function $\mathbf{r}$ whose graph is C is called ________________ the curve C, and $\mathbf{r}$ is called a __________________ of C.

Example 13. Consider $\mathbf{r}_1(t) = \langle \cos(t), \sin(t), t \rangle$ and $\mathbf{r}_2(t) = \langle \cos(2t), \sin(2t), 2t \rangle$, each with domain $[0, 2\pi]$. What do you think the graph of each looks like? How are they similar and how are they different?

Check your intuition

Calculus of vector-valued functions

Unifying theme: Do what you already know, componentwise.

This works with limits:

Example 14. Compute $\lim_{t \to e} \langle t^2, 2, \ln(t) \rangle$.

And with continuity:

Example 15. Determine where the function $\mathbf{r}(t) = t\mathbf{i} - \frac{1}{t^2-4}\mathbf{j} + \sin(t)\mathbf{k}$ is continuous.

And with <u>derivatives</u>:

Example 16. If $\mathbf{r}(t) = \langle 2t - \frac{1}{2}t^2 + 1, t - 1 \rangle$, find $\mathbf{r}'(t)$.

Interpretation: If $\mathbf{r}(t)$ gives the position of an object at time t, then

- $\mathbf{r}'(t)$ gives ______________________________
- $|\mathbf{r}'(t)|$ gives ______________________________
- $\mathbf{r}''(t)$ gives ______________________________

Let's see this graphically

Example 17. Find an equation of the tangent line to $\mathbf{r}(t) = \langle 2t - \frac{1}{2}t^2 + 1, t - 1 \rangle$ at time $t = 2$.