Daily Announcements & Reminders:

Goals for Today: Section 16.3

- Define conservative vector fields and recognize examples from physics
- Learn how to check if a field is conservative
- Compute potential functions
- Apply the Fundamental Theorem of Line Integrals to compute line integrals of conservative vector fields

Strategy for computing normal component line integrals

e.g. flux integrals

1. Find a parameterization $\mathbf{r}(t)$, $a \leq t \leq b$ for the curve C.
2. Compute $x'(t)$ and $y'(t)$ and determine which normal to work with.
3. Substitute: $\int_C \mathbf{F} \cdot \mathbf{n}\, ds = \pm \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \langle y'(t), -x'(t) \rangle\, dt$ (sign based on choice of normal)
4. Integrate

Example 115. Flux of a Velocity Field. Compute the flux of the velocity field $\mathbf{v} = \langle 3 + 2y - y^2/3, 0\rangle$ cm/s across the quarter of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$ in the first quadrant, oriented away from the origin.

Path-Indepedence and Conservative Vector Fields

Definition 116. A vector field $\mathbf{F}$ is **path independent** on an open region D if ____________________________ for all paths C in the region that have the same endpoints.

When $\mathbf{F}$ is path independent, we can use the simplest path from point A to point B to compute a line integral, and will often denote the line integral with points as bounds, e.g.

$$\int_{(0,1,2)}^{(3,1,1)} \mathbf{F} \cdot \mathbf{T}\, ds \qquad \text{or} \qquad \int_{(a,b)}^{(c,d)} \mathbf{F} \cdot d\mathbf{r}.$$

Example 117. If C is any closed path and $\mathbf{F}$ is path independent on a region containing C, then

$$\int_C \mathbf{F} \cdot d\mathbf{r} =$$

Question: Given $\mathbf{F}$, how do we tell if it is path independent on a particular region?

For example, is $\mathbf{F}(x, y) = \langle x, y \rangle$ a path independent vector field on its domain?

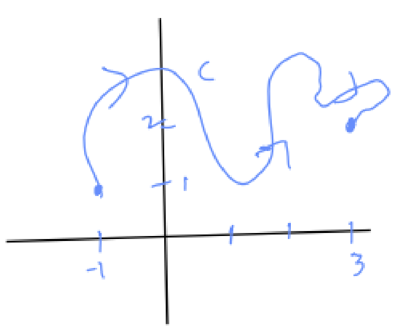
Example 118 (Poll)**.** Last time, we saw that if C is the unit circle about the origin, oriented counterclockwise, then $\int_C \langle -y, x \rangle \cdot d\mathbf{r} = 2\pi$. From this, we can conclude:

A different idea: Suppose $\mathbf{F}$ is a gradient vector field, i.e. $\mathbf{F} = \nabla f$ for some function of multiple variables f. f is called a ______________________ for $\mathbf{F}$. In this case we also say that $\mathbf{F}$ is **conservative.**

Theorem 119 (Fundamental Theorem of Line Integrals)**.** *If C is a smooth curve from the point A to the point B in the domain of a function f with continuous gradient on C, then*

$$\int_C \nabla f \cdot \mathbf{T}\, ds = f(B) - f(A)$$

Example 120. Compute $\int_C \langle x, y \rangle \cdot d\mathbf{r}$ for the curve C shown below from $(-1, 1)$ to $(3, 2)$.



It follows that **every conservative field is path independent.**

In fact, by carefully constructing a potential function, we can show the converse is also true: ________________________________

This leads to a better way to test for path-independence and a way to apply the FToLI.

Curl Test for Conservative Fields: Let $\mathbf{F} = P\mathbf{i}+Q\mathbf{j}+R\mathbf{k}$ be a vector field defined on a **simply-connected** region. If $\operatorname{curl}\mathbf{F} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y\rangle = \langle 0,0,0\rangle$, then $\mathbf{F}$ is conservative.

- If $\mathbf{F}$ is a 2-d vector field, $\operatorname{curl}\mathbf{F} =$
- This is also called the **mixed-partials test**, because

Example 121. Evaluate $\int_C (10x^4 - 2xy^3)\ dx - 3x^2y^2\ dy$ where C is the part of the curve $x^5 - 5x^2y^2 - 7x^2 = 0$ from $(3, -2)$ to $(3, 2)$.