Daily Announcements & Reminders:

Goals for Today: Section 16.2

- Define and explore vector fields
- Define tangential and normal line integrals for vector fields
- Apply vector line integrals to problems involving work, flow, and flux
- Compute vector line integrals using parameterizations

Vector Fields:

Definition 111. A vector field is a function $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^n$ which associates a vector to every point in its domain.

Examples:

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Graphically: For each point (a, b, c) in the domain of $\mathbf{F}$, draw the vector $\mathbf{F}(a, b, c)$ with its base at (a, b, c).

Tools: CalcPlot3d
Field Play

Idea: In many physical processes, we care about the total sum of the strength of that part of a field that lies either in the direction of a curve or perpendicular to that curve.

1. The ______________________ by a field $\mathbf{F}$ on an object moving along a curve C is given by

Example 112. Work Done by a Field. Suppose we have a force field $\mathbf{F}(x, y) = \langle x, y \rangle$ N. Find the work done by $\mathbf{F}$ on a moving object from $(0, 1)$ to $(1, 0)$ in a straight line, where x, y are measured in meters.

1. The ____________________ along a curve C of a velocity field $\mathbf{F}$ for a fluid in motion is given by

When C is __________, this is called _________________. C is called _____________ if it does not intersect itself.

Example 113. Flow of a Velocity Field. Find the circulation of the velocity field $\mathbf{F}(x, y) = \langle -y, x \rangle$ cm/s around the unit circle, parameterized counterclockwise.

Example 114. [Poll] What is the circulation of $\mathbf{F}(x, y) = \langle x, y \rangle$ around the unit circle, parameterized counterclockwise?

Strategy for computing tangential component line integrals

e.g. work, flow, circulation integrals

1. Find a parameterization $\mathbf{r}(t)$, $a \leq t \leq b$ for the curve C.
2. Compute $\mathbf{r}'(t)$.
3. Substitute: $\int_C \mathbf{F} \cdot \mathbf{T}\, ds = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt$
4. Integrate

Idea: ____________ across a plane curve of a 2D-vector field measures the flow of the field across that curve (instead of along it).

We compute this with the integral

$$\int_C \mathbf{F} \cdot \mathbf{n} \, ds.$$

The sign of the flux integral tells us whether the net flow of the field across the curve is in the direction of ________ or in the opposite direction.

We can choose $\mathbf{n}$ to be either of

Example 115. Flux of a Velocity Field. Compute the flux of the velocity field $\mathbf{v} = \langle 3 + 2y - y^2/3, 0 \rangle$ cm/s across the quarter of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$ in the first quadrant.

Strategy for computing normal component line integrals

e.g. flux integrals

1. Find a parameterization $\mathbf{r}(t)$, $a \leq t \leq b$ for the curve C.

2. Compute $x'(t)$ and $y'(t)$ and determine which normal to work with.

3. Substitute: $\int_C \mathbf{F} \cdot \mathbf{n}\ ds = \pm \int_a^b P(\mathbf{r}(t))y'(t) - Q(\mathbf{r}(t))x'(t)\ dt$ (sign based on choice of normal)

4. Integrate