Daily Announcements & Reminders:

Goals for Today: Section 16.1, 16.2

- Define a line integral for a scalar function $f(x, y)$ or $f(x, y, z)$
- Compute line integrals using parameterizations

Unit 4: Vector Calculus

Goals:

- Extend ____________ integrals to ____________ objects living in higher-dimensional space
- Extend the ________________________________ in new ways

We will use tools from everything we have covered so far to do this: parameterizations, derivatives and gradients, and multiple integrals.

Example 105. Suppose we build a wall whose base is the straight line from $(0,0)$ to $(1,1)$ in the xy-plane and whose height at each point is given by $h(x,y) = 2x + y^2$ meters. What is the area of this wall?

Definition 106. The **line integral** of a scalar function $f(x,y)$ over a curve C in $\mathbb{R}^2$ is

$$\int_C f(x,y)\, ds =$$

What things can we compute with this?

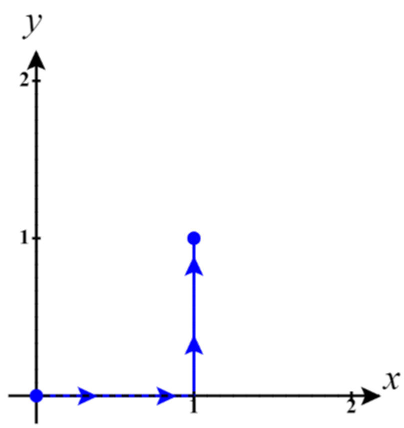
- If $f = 1$:
- If $f = \delta$ is a density function:
- If f is a height:

Strategy for computing line integrals:

1. Parameterize the curve C with some $\mathbf{r}(t)$ for $a \leq t \leq b$
2. Compute $ds = \|\mathbf{r}'(t)\|\ dt$
3. Substitute: $\int_C f(x, y, z)\ ds = \int_a^b f(\mathbf{r}(t))\|\mathbf{r}'(t)\|\ dt$
4. Integrate

Example 107. [Poll] Compute $\int_C 2x + y^2\ ds$ along the curve C given by $\mathbf{r}(t) = 10t\mathbf{i} + 10t\mathbf{j}$ for $0 \leq t \leq \frac{1}{10}$.

Example 108. Compute $\int_C 2x + y^2 \; ds$ along the curve C pictured below.



Example 109 (Poll)**.** Let C be a curve parameterized by $\mathbf{r}(t)$ from $a \leq t \leq b$. Select all of the true statements below.

a) $\mathbf{r}(t+4)$ for $a \leq t \leq b$ is also a parameterization of C with the same orientation

b) $\mathbf{r}(2t)$ for $a/2 \leq t \leq b/2$ is also a parameterization of C with the same orientation

c) $\mathbf{r}(-t)$ for $a \leq t \leq b$ is also a parameterization of C with the opposite orientation

d) $\mathbf{r}(-t)$ for $-b \leq t \leq -a$ is also a parameterization of C with the opposite orientation

e) $\mathbf{r}(b-t)$ for $0 \leq t \leq b-a$ is also a parameterization of C with the opposite orientation

Example 110. Find a parameterization of the curve C that consists of the portion of the curve $y = x^2+1$ from $(1, 2)$ to $(0, 1)$ and use it to write the integral $\int_C x^2+y^2\, ds$ as an integral with respect to your parameter.