Daily Announcements & Reminders:

Goals for Today: Section 15.8

- Change variables in multiple integrals
- Identify choices for changing variables in a given integration problem

Thinking about single variable calculus: Compute $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\, dx$

Theorem 100 (Substitution Theorem)**.** *Suppose* $\mathbf{T}(u, v)$ *is a one-to-one, differentiable transformation that maps the region* G *in the* uv*-plane to the region* R *in the* xy*-plane. Then*

$$\iint_R f(x, y)\ dx\ dy = \iint_G f(\mathbf{T}(u, v))|\det(D\mathbf{T}(u, v))|\ du\ dv.$$

Example 101. Evaluate $\displaystyle\int_0^4 \int_{y/2}^{y/2+1} \frac{2x - y}{2}\ dx\ dy$ via the transformation $x = u + v$, $y = 2v$.

1. **Find T:**

2. **Find G and sketch:**

3. **Find Jacobian:**

4. **Convert and use theorem:**

Example 102. a) [Poll] Find the Jacobian of the transformation

$$x = u + (1/2)v, \ y = v.$$

b) [Poll] Which transformation(s) seem suitable for the integral

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3(2x-y)e^{(2x-y)^2} \, dx \, dy?$$

i) $u = x, v = y$

ii) $u = \sqrt{x^2+y^2}, v = \arctan(y/x)$

iii) $u = 2x - y, \ v = y^3$

iv) $u = y, \ v = 2x - y$

v) $u = 2x - y, \ v = y$

vi) $u = e^{(2x-y)^2}, \ v = y^3$

Theorem 103 (Derivative of Inverse Coordinate Transformation)**.** *If* $\mathbf{T}(u,v)$ *is a one-to-one differentiable transformation that maps a region* G *in the* uv*-plane to a region* R *in the* xy*-plane and* $T(u_0,v_0) = (x_0,y_0)$*, then we have*

$$|\det(D\mathbf{T}(u_0,v_0))| = \frac{1}{|\det(D\mathbf{T}^{-1}(x_0,y_0))|}$$

Example 104. Let's evaluate $\iint_R \frac{y(x+y)}{x^3}$ where R is the region in the xy-plane bounded by $y = x, y = 3x, y = 1 - x$, and $y = 2 - x$. Consider the coordinate transformation $u = x + y, v = y/x$.

1. **Find the rectangle G in the uv plane that is mapped to R**

2. **Evaluate $f(\mathbf{T}(u, v))|\det(D\mathbf{T}(u, v))|$ in terms of u and v without directly solving for $\mathbf{T}$ using the theorem above**

3. **Use the Substitution Theorem to compute the integral.**