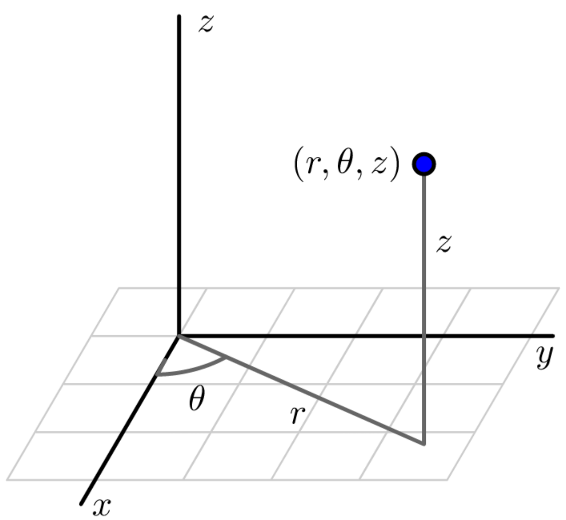
Daily Announcements & Reminders:

Goals for Today: Section 15.7

- Be able to convert between Cartesian, cylindrical, and spherical coordinate systems in $\mathbb{R}^3$
- Compute triple integrals expressed in cylindrical coordinates
- Compute triple integrals expressed in spherical coordinates

Cylindrical Coordinate System



Cylindrical to Cartesian:

$$x = r\cos(\theta), \quad y = r\sin(\theta), \quad z = z$$

Cartesian to Cylindrical:

$$r^2 = x^2 + y^2, \quad \tan(\theta) = \frac{y}{x}, \quad z = z$$

For uniqueness:

Example 92. a) Find cylindrical coordinates for the point with Cartesian coordinates $(-1, \sqrt{3}, 3)$.

b) Find Cartesian coordinates for the point with cylindrical coordinates $(2, 5\pi/4, 1)$.

Example 93. In xyz-space sketch the *cylindrical box*

$$B = \{(r, \theta, z) \mid 1 \le r \le 2,\ \pi/6 \le \theta \le \pi/3,\ 0 \le z \le 2\}.$$

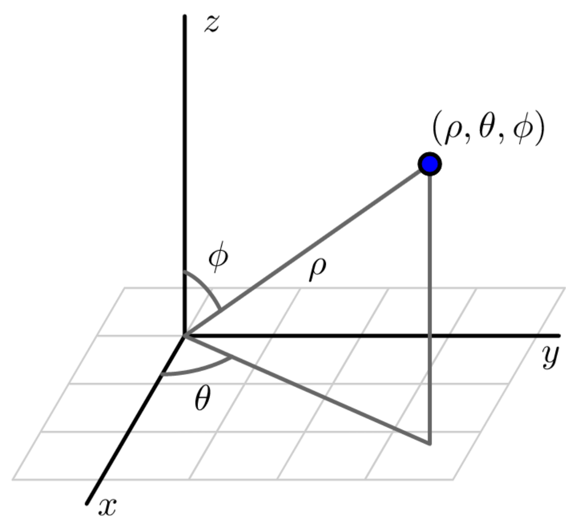
Triple Integrals in Cylindrical Coordinates

We have $dV =$ ____________________

Example 94. Set up a iterated integral in cylindrical coordinates for the volume of the region D lying below $z = x + 2$, above the xy-plane, and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Example 95 (Poll)**.** Suppose the density of the cone defined by $r = 1 - z$ with $z \geq 0$ is given by $\delta(r, \theta, z) = z$. Set up an iterated integral in cylindrical coordinates that gives the mass of the cone.

Spherical Coordinate System



Spherical to Cartesian:

$$x = \rho \sin(\varphi) \cos(\theta)$$
$$y = \rho \sin(\varphi) \sin(\theta)$$
$$z = \rho \cos(\varphi)$$

Cartesian to Spherical:

$$\rho^2 = x^2 + y^2 + z^2$$
$$\tan(\theta) = \frac{y}{x}$$
$$\tan(\varphi) = \frac{\sqrt{x^2 + y^2}}{z}$$

For uniqueness:

Example 96. a) Find spherical coordinates for the point with Cartesian coordinates $(-2, 2, \sqrt{8})$.

b) Find Cartesian coordinates for the point with spherical coordinates $(2, \pi/2, \pi/3)$.

Example 97. In xyz-space sketch the *spherical box*

$$B = \{(\rho, \varphi, \theta) \mid 1 \le \rho \le 2,\ 0 \le \varphi \le \pi/4,\ \pi/6 \le \theta \le \pi/3\}.$$

Triple Integrals in Spherical Coordinates

We have $dV =$ ______________________________

Example 98. Write an iterated integral for the volume of the "ice cream cone" D bounded above by the sphere $x^2+y^2+z^2 = 1$ and below by the cone $z = \sqrt{3}\sqrt{x^2+y^2}$.

Example 99 (Poll)**.** Write an iterated integral for the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cylinder $x^2 + y^2 = 1$.