Daily Announcements & Reminders:

Goals for Today: Sections 15.1, 15.2

- Introduce double and iterated integrals for functions of two variables on rectangles
- Use Fubini's Theorem to change the order of integration of a iterated integral
- Be able to set up & evaluate a double integral over a general region
- Change the order of integration for general regions

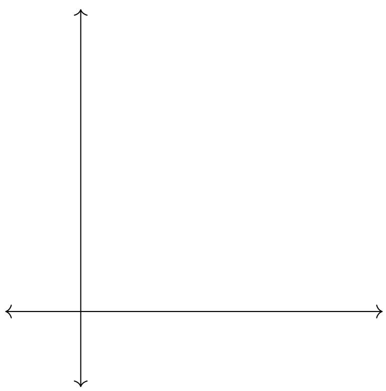
Recall: Riemann sum and the definite integral from single-variable calculus.

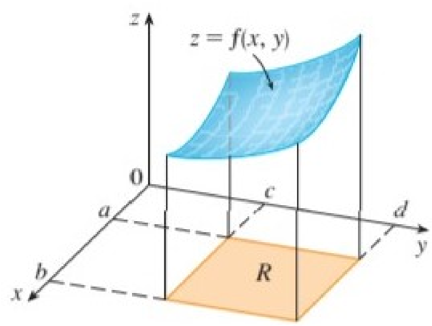
Double Integrals

Volumes and Double integrals Let R be the closed rectangle defined below:

$$R = [a, b] \times [c, d] = \{(x, y) \in \mathbb{R}^2 | a \leq x \leq b, c \leq y \leq d\}$$

Let $f(x, y)$ be a function defined on R such that $f(x, y) \geq 0$. Let S be the solid that lies above R and under the graph f.





Question: How can we estimate the volume of S?

Definition 73. The ________________________ of $f(x, y)$ over a rectangle R is

$$\iint_R f(x, y)\ dA = \lim_{|P| \to 0} \sum_{k=1}^{n} f(x_k, y_k) \Delta A_k$$

if this limit exists.

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Question: How can we compute a double integral?

Answer:

Suppose that f is a function of two variables that is integrable on the rectangle $R = [a, b] \times [c, d]$.

What does $\int_c^d f(2, y) dy$ represent?

What about $\int_c^d f(x,y)dy$?

Let $A(x) = \int_c^d f(x,y)dy$. Then,

$$= \int_a^b A(x)dx =$$

This is called an ____________________.

Example 74. Evaluate $\displaystyle\int_1^2 \int_3^4 6x^2y \; dy \; dx.$

Theorem 75 (Fubini's Theorem)**.** *If f is continuous on the rectangle $R = [a,b] \times [c,d]$, then*

More generally, this is true if we assume that f is bounded on R, f is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

Example 76. Compute $\displaystyle\iint_R xe^{e^{e^y}}\, dA$, where R is the rectangle $[-1, 1] \times [0, 4]$.

Question: What if the region R we wish to integrate over is not a rectangle?

Answer: Repeat same procedure - it will work if the boundary of R is smooth and f is continuous.

Example 77. Compute the volume of the solid whose base is the triangle with vertices $(0,0),(0,1),(1,0)$ in the xy-plane and whose top is $z = 2 - x - y$.

Vertically simple:

Horizontally simple: