Daily Announcements & Reminders:

Goals for Today: Sections 14.4-14.6

- Learn to compute the rate of change of a multivariable function in any direction
- Investigate the connection between the gradient vector and level curves/surfaces
- Discuss tangent planes to surfaces, how to find them, and when they exist

Example 66. Recall that if $z = f(x, y)$, then f_x represents the rate of change of z in the x-direction and f_y represents the rate of change of z in the y-direction. What about other directions?

Let's go back to our hill example again, $h(x, y) = 4 - \frac{1}{4}x^2 - \frac{1}{4}y^2$. How could we figure out the rate of change of our height from the point $(2, 1)$ if we move in the direction $\langle -1, 1 \rangle$?

Definition 67. The ____________________________ of $f : \mathbb{R}^n \to \mathbb{R}$ at the point $\mathbf{p}$ in the direction of a unit vector $\mathbf{u}$ is

$$D_{\mathbf{u}} f(\mathbf{p}) =$$

if this limit exists.

E.g. for our hill example above we have:

Note that $D_{\mathbf{i}} f =$ $D_{\mathbf{j}} f =$ $D_{\mathbf{k}} f =$

Definition 68. If $f : \mathbb{R}^n \to \mathbb{R}$, then the ________________ of f at $\mathbf{p} \in \mathbb{R}^n$ is the vector function ____ (or ________) defined by

$$\nabla f(\mathbf{p}) =$$

Note: If $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable at a point $\mathbf{p}$, then f has a directional derivative at $\mathbf{p}$ in the direction of any unit vector $\mathbf{u}$ and

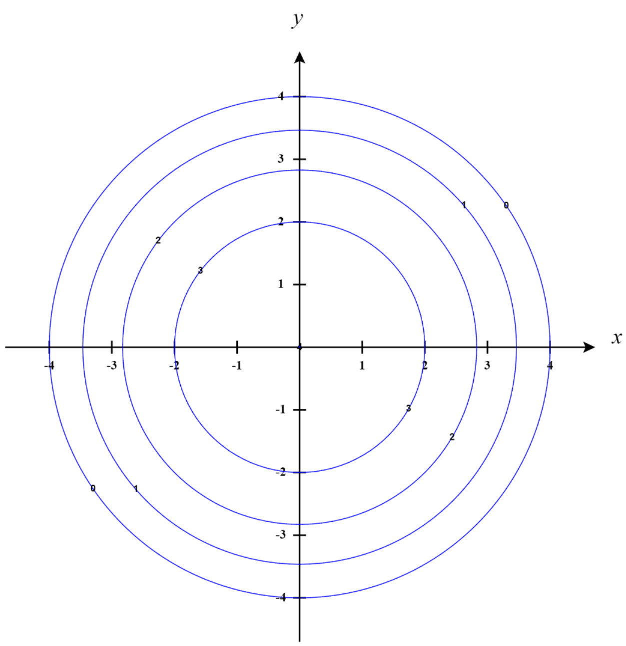
$$D_{\mathbf{u}} f(\mathbf{p}) =$$

Example 69. Find the gradient vector and the directional derivative of each function at the given point $\mathbf{p}$ in the direction of the given vector $\mathbf{u}$.

a) $f(x, y) = \ln(x^2 + y^2), \mathbf{p} = (-1, 1), \mathbf{u} = \left\langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle$

b) $g(x, y, z) = x^2 + 4xy^2 + z^2$, $\mathbf{p} = (1, 2, 1)$, $\mathbf{u}$ the unit vector in the direction of $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

Example 70. If $h(x, y) = 4 - \frac{1}{4}x^2 - \frac{1}{4}y^2$, the contour map is given below. Find and draw ∇h on the diagram at the points $(2, 0), (0, 4)$, and $(-\sqrt{2}, -\sqrt{2})$. At the point $(2, 0)$, compute $D_{\mathbf{u}}h$ for the vectors $\mathbf{u}_1 = \mathbf{i}, \mathbf{u}_2 = \mathbf{j}, \mathbf{u}_3 = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$.

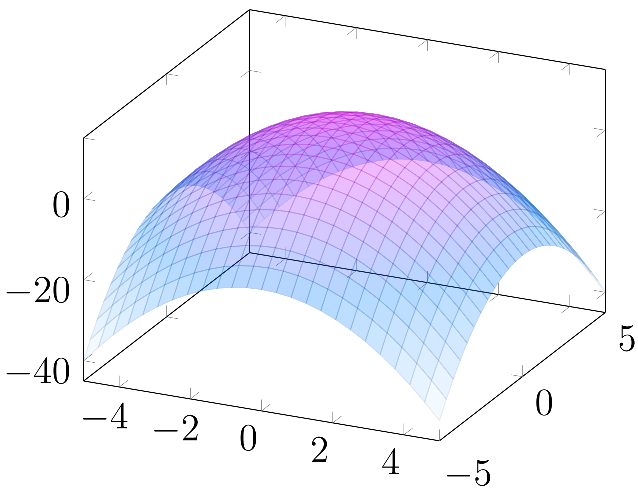


Note that the gradient vector is ________________ to level curves.

Similarly, for $f(x, y, z)$, $\nabla f(a, b, c)$ is ____

Tangent planes to level surfaces

Suppose S is a surface with equation $F(x, y, z) = k$. How can we find an equation of the tangent plane of S at $P(x_0, y_0, z_0)$?



$x^2 + y^2 + z = 10, P = (-1, 3, 0)$

Example 71. Find the equation of the tangent plane at the point $(-2, 1, -1)$ to the surface given by

$$z = 4 - x^2 - y$$

Special case: if we have $z = f(x, y)$ and a point $(a, b, f(a, b))$, the equation of the tangent plane is

This should look familiar: it's ______________________________