

No books or notes may be used on this exam. **Choose one of the two problems below to solve.** You will have 45 minutes to answer your chosen question. Please write legibly and keep your paper as organized as possible. I will collect both your scratch work and your final answer. **Your final answers should be nicely written to convey your ideas and convince the reader of the answer.**

Turn off your cell phone and do not wear headphones during the exam. You may use a calculator as long as it does not connect to the internet. Good Luck!

Name: \_\_\_\_\_

**The Game of Crosses:** The following game is played with a partner. Start with  $n$  crosses. Players alternate connecting any two free ends, without intersecting any drawn edges, and then draw a dash in the new edge to create another cross. The winner is the player that makes the last move; when there is no way to make a new edge without intersecting a previously drawn edge. Is there a winning strategy and if so which player will win? (Your answer may depend on the value of  $n$ ).

**Crossing out Circles:** Jacqueline writes out the numbers 1, 2, 3, and 4 in a circle. Starting at 1, she crosses out every second integer until just one number remains: 2 is crossed out first, then 4, leaving 1 and 3. As she continues around the circle, 3 is crossed out next, leaving 1 as the last number left.

Suppose Jacqueline instead writes out the numbers  $1, 2, 3, 4, \dots, n$  in a circle.

For what values of  $n$  will the number 1 be the last number left?

**Solution:**

**Problem Chosen:** \_\_\_\_\_