MA114 Summer II 2017
Worksheet – Direction Fields & Euler's Method
7/27/17

1. Sketch the slope field of the differential equation. Then use it to sketch a solution curve that passes through the given point.

   a) $y' = y - 2x, (1, 0)$

   b) $y' = xy - x^2, (0, 1)$

2. Let $F(t, y) = t^2 - y$ and let $y(t)$ be the solution of $y' = F(t, y)$ satisfying $y(2) = 3$. Let $h = 0.1$ be the time step in Euler's Method and set $y_0 = y(2) = 3$.

   a) Calculate $y_1 = y_0 + hF(2, 3)$.

   b) Calculate $y_2 = y_1 + hF(2.1, y_1)$.

   c) Calculate $y_3 = y_2 + hF(2.2, y_2)$ and continue computing y_4, y_5, and y_6.

   d) Find approximations to $y(2.2)$ and $y(2.5)$.

3. Consider the autonomous differential equation $y' = y^2(3 - y)(y + 1)$.

   a) Find the equilibrium values of y.

   b) Sketch the phase portrait (direction field) of this differential equation, paying particular attention to the behavior near equilibria.

   c) Classify each equilibrium point as *stable* or *unstable*.

4. Consider the autonomous differential equation from the previous problem. Without solving the differential equation, determine the value of $\lim_{y\to\infty} y(t)$, where the initial value is

   a) $y(0) = 1$

   b) $y(0) = 4$

   c) $y(0) = -4$

5. Sketch the direction field for $y' = y^2$.

6. Use Euler's method with step size 0.5 to compute the approximate y-values y_1, y_2, y_3, and y_4 of the solution of the initial value problem $y' = y - 2x, y(1) = 0$.