MA114 Summer II 2017
Worksheet – Differential Equations
7/26/17

1. a) Is $y = \sin(3x) + 2e^{4x}$ a solution to the differential equation $y'' + 9y = 50e^{4x}$? Explain why or why not.

   b) Explain why every solution of $dy/dx = y^2 + 6$ must be an increasing function.

   c) What does it mean to say that a differential equation is linear or nonlinear?

2. Find all values of α so that $y(x) = e^{\alpha x}$ is a solution of the differential equation $y'' + y' - 12y = 0$.

3. A tank has pure water flowing into it at 10 liters/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 liters/min. Salt is added to the tank at the rate of 0.1 kg/min. Initially, the tank contains 10 kg of salt in 100 liters of water. Formulate an initial value problem (that is, a differential equation along with initial conditions) whose solution is the quantity of salt in the tank at any time t. Do not solve the initial value problem.

4. (Adapted from Boyce and DiPrima, Section 2.3 # 19) Consider a lake of constant volume V containing at time t an amount $Q(t)$ of pollutant, evenly distributed throughout the lake with a concentration $c(t)$, where $c(t) = Q(t)/V$. Assume that (i) water containing concentration k of pollutant enters the lake at a rate r, (ii) water leaves the lake at the same rate r, (iii) pollutants are also added directly to the lake at a rate P. Note that many factors, including precipitation, absorption, evaporation, heterogeneity in lake temperature, etc., are being neglected. Assuming the initial concentration of pollutant is c_0, formulate an initial value problem whose solution is the concentration, $c(t)$, of pollutant at any time t. Do not solve.

5. Show that for any value of c, $y = x^2 + \dfrac{c}{x^2}$ is a solution to the differential equation $xy' + 2y = 4x^2, (x > 0)$. Find the value of c for which the solution satisfies the initial condition $y(6) = 5$.