MA114 Summer II 2017
Worksheet 8 – Exam 1 Review Solutions
6/21/17

1. Compute the following integrals.

a) $\int \sqrt{4-9x^2}\,dx$,

b) $\int \sin^2(x)\sin(2x)\cos(x)\,dx$
Hint: $\sin(2x) = 2\sin(x)\cos(x)$,

c) $\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right)dx$,

d) $\int \tan^3(y)\sec^2(y)dy$,

e) $\int 6x^2 \cdot \frac{\cos(2x^3-5)}{\sin(2x^3-5)}\,dx$,

f) $\int \frac{dx}{\sqrt{6-x^2}}$,

g) $\int \frac{(1-x^2)x}{(1+x^2)^2}\,dx$,

h) $\int \frac{1}{x^2-x}\,dx$.

Solution to 1.

a) $\frac{1}{2}x\sqrt{4-9x^2} + \frac{2}{3}\arcsin(3x/2) + C$,

b) $\frac{-2}{3}\cos^3(x) + \frac{2}{5}\cos^5(x) + C$,

c) $\frac{\sqrt{3}}{6}\pi - \frac{\pi}{4} + \frac{1}{2}\ln(2) + C$,

d) $\frac{1}{4}\tan^4(y) + C$,

e) $\ln|\sin(2x^3-5)| + C$,

f) $\arcsin\left(\frac{x}{\sqrt{6}}\right) + C$,

g) $-\frac{1}{1+x^2} - \frac{1}{2}\ln|1+x^2| + C$,

h) $\ln|x-1| - \ln|x| + C$.

2. Sketch the graph of the function $f(x) = x^2$ on $[0,4]$ as well as the approximations L_4, R_4, M_4, and T_4. Compute each approximation.

Solution to 2. $L_4 = 0^2+1^2+2^2+3^2 = 14$. $R_4 = 1^2+2^2+3^2+4^2 = 30$. $M_4 = 0.5^2+1.5^2+2.5^2+3.5^2 = 21$. $T_4 = 1/2(L_4+R_4) = 22$.

3. Determine the form of the partial fraction decompositions for each of the following fractions.

a) $\frac{2x+1}{(x-1)(x^2+1)^2}$

b) $\frac{x^2+1}{(x^2-1)(x+3)^2}$

Solution to 3.

a) $\frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$

b) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3} + \frac{D}{(x+3)^2}$

4. Determine whether the following improper integrals are convergent or divergent. Explain your reasoning

a) $\displaystyle\int_1^{\infty} \frac{8}{(2x+3)^3}\, dx$

b) $\displaystyle\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x}}\, dx$

Solution to 4.

a) The integral converges by comparison to $\int_1^{\infty} \frac{8}{x^3} dx$ which converges since this is a p-integral with $p = 3 > 1$.

b) The integral diverges by comparison to $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$, which diverges since it is a p-integral with $p = 1/2 \leq 1$.

5. Determine whether the following improper integrals are convergent or divergent. If the integral converges, find the value it converges to.

a) $\displaystyle\int_0^{-1} \frac{e^{1/x}}{x^3}\, dx$

b) $\displaystyle\int_0^{2} \frac{x}{x-1}\, dx$

Solution to 5.

a) The integral converges to $2/e$.

b) The integral diverges since $\displaystyle\lim_{t\to 1^-} \frac{1}{t-1}$ diverges.