MA114 Summer 2018
Final Exam Review

1. Find the volume of the following solids.

   a) The solid obtained by rotating the region bounded by $y = x^2$ and $x = y^2$ about the x-axis,

   b) The solid obtained by rotating the region bounded by $x = y^2$ and $x = 1$ about the line $x = 1$

   c) The solid whose base is the right half of the unit circle and whose cross-sections perpendicular to the x-axis are squares.

2. Find the area of the surface of revolution obtained by rotating the given curve about the given axis: $y = \sqrt{x+1}, 0 \leq x \leq 3$, about the x-axis

3. Set up but do not solve an integral which computes the surface area obtained by rotating the curve $y = x^3, 0 \leq x \leq 2$, about the x-axis.

4. Compute the arc length of the following curves

   a) $x = a\cos^3\theta, y = a\sin^3\theta, 0 \leq \theta \leq 2\pi$,

   b) $y = \sqrt{2 - x^2}, 0 \leq x \leq 1$,

   c) $r = 2\cos(\theta), 0 \leq \theta \leq 2\pi$.

   d) $x = 3t^2, y = 2t^3, 0 \leq t \leq 5$.

5. Find the centroid of the region bounded by $y = \sqrt{x}$ and $y = x$.

6. Compute the slope of the tangent line to the curve in Problem 4(a) above, with $a = 8$, at the point $(1, 3\sqrt{3})$. Use this to determine an equation for the tangent line.

7. Find the slope of the tangent line to the curve $r = 2\cos\theta$ at $\theta = \pi/3$.

8. Compute the area of the following regions.

   a) The region enclosed by the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.

   b) The region enclosed by one loop of the rose $r = 8\cos(5\theta)$.

9. Convert the Cartesian point to polar coordinates or vice versa.

   a) $(x, y) = (\sqrt{3}, 1)$

   b) $(x, y) = (-4, 4)$

   c) $(r, \theta) = (2, 3\pi/2)$

   d) $(r, \theta) = (-3, -\pi/3)$

10. By converting to Cartesian coordinates, identify and graph the curve $r^2 \sin 2\theta = 1$ (It may help to remember the identity $\sin 2\theta = 2 \sin\theta \cos\theta$.)

11. Is $y(x) = 7e^{4x} + 2e^{-3x}$ a solution to the differential equation $y'' - y' - 12y = 0$?

12. Draw a direction field for the differential equation $y' = y(1-y)$. What are the equilibria? Classify each as stable or unstable. If a solution satisfies $y(0) = 0.5$, what is $\lim_{x\to\infty} y(x)$?

13. Suppose that a population $P(t)$ grows according to the model

$$\frac{dP}{dt} = 0.4P - 0.001P^2, P(0) = 50.$$

    a) What is the carrying capacity? (the maximum sustainable population)
    b) What is $P'(0)$?
    c) When will the population reach 50% of the carrying capacity?

14. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x \sin x}{y}, \quad y(0) = -1.$$

15. Find the general solution to the differential equations:

    a) $y' = x - y$
    b) $ty' - 2y = y^2, t > 0$
    c) $y' + 2xy = 1$

16. Find the points (x, y) where the slope of the tangent line to the parametric curve $x = \cos^3 t, y = \sin^3 t$ is

    a) 0,
    b) undefined,
    c) -1.

17. Find the area of one petal of the "four-petal rose" $r = \sin(2\theta)$. Then show that the total area of the rose is half of the area of the circumscribed circle.